Circle your TA discussion section.

- AD1, TR 11:00-12:50, Adriana Morales  
- AD2, TR 9:00-10:50, Hannah Burson  
- AD3, TR 1:00-2:50, Dana Neidinger  
- ADA, TR 8:00-8:50, Gayana Jayasinghe  
- ADB, TR 9:00-9:50, Felix Clemen  
- ADC, TR 10:00-10:50, Lutian Zhao  
- ADD, TR 11:00-11:50, Gidon Orelowitz  
- ADE, TR 12:00-12:50, Josh Wen  
- ADF, TR 1:00-1:50, Nachiketa Adhikari  
- ADG, TR 2:00-2:50, Lutian Zhao  
- ADH, TR 3:00-3:50, Stathis Chronstios  
- ADI, TR 4:00-4:50, Stathis Chronstios

- ADJ, TR 9:00-9:50, Gayana Jayasinghe  
- ADK, TR 10:00-10:50, Madina Bolat  
- ADL, TR 11:00-11:50, Chris Loa  
- ADM, TR 12:00-12:50, Heeyeon Kim  
- ADN, TR 1:00-1:50, Josh Wen  
- ADO, TR 2:00-2:50, Kesav Krishnan  
- ADQ, TR 10:00-10:50, Felix Clemen  
- ADR, TR 9:00-9:50, Madina Bolat  
- ADS, TR 12:00-12:50, Chris Loa  
- ADT, TR 2:00-2:50, Nachiketa Adhikari  
- ADU, TR 3:00-3:50, Kesav Krishnan  
- ADZ, TR 9:00-9:50, Gidon Orelowitz

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quickly turn in your test to me or a TA and show your Student ID.

FRONT OF ROOM - 114 David Kinley Hall
1. (10 points) Find \( w'(x) \) given that \( w(x) = 5x^9e^{12x} \)

\[
w'(x) = \frac{d}{dx}(5x^9e^{12x}) = 45x^8e^{12x} + 5x^9 \cdot 12e^{12x} = 45x^8e^{12x} + 60x^9e^{12x}
\]

2. (10 points) Find \( g'(x) \) given that \( g(x) = \frac{x^8 + 3\cos(x)}{x^4 + 5\sin(x)} \)

\[
g'(x) = \frac{d}{dx} \left( \frac{x^8 + 3\cos(x)}{x^4 + 5\sin(x)} \right) = \frac{(8x^7 - 3\sin(x))(x^4 + 5\sin(x)) - (x^8 + 3\cos(x))(4x^3 + 5\cos(x))}{(x^4 + 5\sin(x))^2}
\]
3. (10 points) Find $f'(x)$ given that $f(x) = \cot (\ln (\sec (5x)))$

$$f'(x) = -\csc^2 (\ln (\sec (5x))) \cdot \frac{d}{dx} (\ln (\sec (5x)))$$

$$f'(x) = -\csc^2 (\ln (\sec (5x))) \cdot \frac{1}{\sec (5x)} \cdot \frac{d}{dx} (\sec (5x))$$

$$f'(x) = -\csc^2 (\ln (\sec (5x))) \cdot \frac{1}{\sec (5x)} \cdot \sec (5x) \tan (5x) \cdot \frac{d}{dx} (5x)$$

$$f'(x) = -\csc^2 (\ln (\sec (5x))) \cdot \frac{1}{\sec (5x)} \cdot \sec (5x) \tan (5x) \cdot 5$$

4. (10 points) Find $\frac{dy}{dx}$ and write your answer in terms of $x$ given the function $y = (8x)^{\frac{9}{x}}$

$$y = (8x)^{\frac{9}{x}} = e^{\ln ((8x)^{\frac{9}{x}})} = e^{\frac{9}{x} \ln (8x)}$$

$$\frac{dy}{dx} = e^{\frac{9}{x} \ln (8x)} \cdot \frac{d}{dx} \left( \frac{9}{x} \ln (8x) \right)$$

$$\frac{dy}{dx} = e^{\frac{9}{x} \ln (8x)} \cdot \frac{9 \cdot \ln (8x) \cdot (1) - (9 \ln (8x)) \cdot (1)}{(x)^2}$$

$$\frac{dy}{dx} = e^{\frac{9}{x} \ln (8x)} \cdot \frac{9 - 9 \ln (8x)}{(x)^2}$$

$$\frac{dy}{dx} = (8x)^{\frac{9}{x}} \cdot \frac{9 - 9 \ln (8x)}{(x)^2}$$
5. (10 points) Find the slope of the line tangent to the curve \( x^4 y^2 = 75x - 2y \) at the point (2, 3).

\[
\frac{dy}{dx} = \frac{\frac{d}{dx} (x^4 y^2)}{\frac{d}{dx} (75x - 2y)} = \frac{d}{dx} (75x - 2y)
\]

\[
4x^3 y^2 + x^4 \cdot \frac{dy}{dx} y^2 = 75 - 2 \cdot \frac{dy}{dx}
\]

\[
2x^4 y \frac{dy}{dx} + 2x^4 y^2 = 75 - 4x^3 y^2
\]

\[
\frac{dy}{dx} = \frac{75 - 4x^3 y^2}{2x^4 y + 2}
\]

\[
\left. \frac{dy}{dx} \right|_{(x, y) = (2, 3)} = \frac{75 - 4(2)^3 (3)^2}{2(2)^4 (3) + 2} = \frac{-2}{98}
\]

6. (10 points) Evaluate the following limit. Simplify your answer.

\[
\lim_{x \to 0} \frac{e^{5x} - 5x - 1}{1 - \cos(9x)} = \lim_{x \to 0} \frac{5e^{5x} - 5}{9\sin(9x)}
\]

\[
= \lim_{x \to 0} \frac{25e^{5x}}{8\cos(9x)} = \frac{25}{81}
\]
7. (10 points) Determine the $x$-coordinate for the absolute minimum value of the following function.

\[ f(x) = 2 \ln (64x^2 + 1) - 320 \arctan (8x) \]

\[ f'(x) = 2 \cdot \frac{1}{64x^2 + 1} \cdot 128x - 320 \cdot \frac{1}{(8x)^2 + 1} \cdot 8 \]

\[ = \frac{256x}{64x^2 + 1} - \frac{2560}{64x^2 + 1} \]

\[ = \frac{256x - 2560}{64x^2 + 1} \]

\[ = \frac{256(x - 10)}{64x^2 + 1} \]

Values of $f'(x)$

\[ --- 0 + + + \]

$f$ decreases 10 to infinity

Absolute min at $x = 10$

\[ f(10) = 2 \ln (64(10)^2 + 1) - 320 \arctan (80) \]
8. (10 points) A function \( f(x) \) is differentiable everywhere and has the following second derivative.

\[
f''(x) = \frac{(2x^2 - 288)(x + 3)^2(x^2 + 25)}{20e^{16-x}}.
\]

Find the intervals of concavity for \( f(x) \) and state each \( x \)-value at which the graph of \( f(x) \) has an inflection point.

\[
\begin{array}{c|c|c|c|c|c}
\text{Values of } f''(x) & + & 0 & - & 0 & + \\
\hline
-12 & 1 & -3 & 12 \\
\end{array}
\]

- \( f \) concave up: \( (-\infty, -12) \) \( (12, \infty) \)
- \( f \) concave down: \( (-12, -3) \) \( (-3, 12) \) or \( (-12, 12) \)

Inflection points at these \( x \)-values:
- \( x = -12 \)
- \( x = 12 \)

9. (10 points) The curve \( y = f(x) \) has the property that the slope of the curve is always equal to its \( y \)-coordinate multiplied by \( 1/4 \). If the curve goes through the point \( (\ln(81), 36) \), then find a formula for \( f(x) \). Simplify your answer.

\[
\frac{dy}{dx} = \frac{1}{4}y \implies y = Ce^{\frac{1}{4}x}
\]

\[
36 = Ce^{\frac{1}{4}\ln(81)}
\]

\[
C = \frac{36}{e^{\frac{1}{4}\ln(81)}} = \frac{36}{e^{\ln(81)^{1/4}}} = \frac{36}{e^{\ln(3)}} = \frac{36}{3} = 12
\]

\[
y = 12e^{\frac{1}{4}x}
\]
10. (10 points) A spherical balloon is being inflated so that its diameter is increasing at a constant rate of 6 cm/min. How quickly is the volume of the balloon increasing when the diameter is 50 cm?

\[ D = \text{diameter } (D = 2r) \]

Given \( \frac{dD}{dt} = 6 \text{ cm/min} \)

Want \( \frac{dV}{dt} \bigg| _{D=50\text{ cm}} \)

\[ V = \frac{4}{3} \pi r^3 \]

\[ V = \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 \]

\[ V = \frac{\pi}{6} D^3 \]

\( \frac{d}{dt} (V) = \frac{d}{dt} \left( \frac{\pi}{6} D^3 \right) \)

\( \frac{dV}{dt} = \frac{\pi}{2} D^2 \frac{dD}{dt} \)

\( \frac{dV}{dt} = \frac{\pi}{2} (50)^2 \cdot (6) \)

\( \frac{dV}{dt} = 7500\pi \text{ cm}^3/\text{min} \)
Students – do not write on this page!

1. (10 points) ______________________

2. (10 points) ______________________

3. (10 points) ______________________

4. (10 points) ______________________

5. (10 points) ______________________

6. (10 points) ______________________

7. (10 points) ______________________

8. (10 points) ______________________

9. (10 points) ______________________

10. (10 points) _____________________

TOTAL (100 points) _______________