Circle your TA discussion section.

- AD1, TR 11:00-12:50, Adriana Morales
- AD2, TR 9:00-10:50, Hannah Burson
- AD3, TR 1:00-2:50, Dana Neidinger
- ADA, TR 8:00-8:50, Gayana Jayasinghe
- ADB, TR 9:00-9:50, Felix Clemen
- ADC, TR 10:00-10:50, Lutian Zhao
- ADD, TR 11:00-11:50, Gidon Orelowitz
- ADE, TR 12:00-12:50, Josh Wen
- ADF, TR 1:00-1:50, Nachiketa Adhikari
- ADG, TR 2:00-2:50, Lutian Zhao
- ADH, TR 3:00-3:50, Stathis Chrontsios
- ADI, TR 4:00-4:50, Stathis Chrontsios
- ADJ, TR 9:00-9:50, Gayana Jayasinghe
- ADK, TR 10:00-10:50, Madina Bolat
- ADL, TR 11:00-11:50, Chris Loa
- ADM, TR 12:00-12:50, Heeyeon Kim
- ADN, TR 1:00-1:50, Josh Wen
- ADO, TR 2:00-2:50, Kesav Krishnan
- ADQ, TR 10:00-10:50, Felix Clemen
- ADR, TR 9:00-9:50, Madina Bolat
- ADS, TR 12:00-12:50, Chris Loa
- ADT, TR 2:00-2:50, Nachiketa Adhikari
- ADU, TR 3:00-3:50, Kesav Krishnan
- ADZ, TR 9:00-9:50, Gidon Orelowitz

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.
1. (7 points each) Evaluate the following limits and write your answers in simplified form. For infinite limits, you must clearly show whether the limit is \( \infty \) or \(-\infty\). We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

(a) \[
\lim_{x \to 2} \frac{x^2 - 3x - 10}{x^7 - 16x^3} \quad \rightarrow \quad 0 \quad \rightarrow \quad \frac{(x-5)(x+2)}{x^3(x^4-16)}
\]
\[
= \lim_{x \to 2} \frac{(x-5)(x+2)}{x^3(x^2+4)(x^2-4)}
\]
\[
= \lim_{x \to 2} \frac{(x-5)(x+2)}{x^3(x^2+4)(x-2)(x+2)}
\]
\[
= \lim_{x \to 2} \frac{x-5}{x^3(x^2+4)(x-2)}
\]
\[
= \frac{-7}{256}
\]

(b) \[
\lim_{x \to 9^+} \frac{14-x}{4-\sqrt{x+7}}
\]
\[
\lim_{x \to 9^+} \frac{14-x}{4-\sqrt{x+7}} \rightarrow 5 \quad \rightarrow \quad -\infty
\]

Note: \( x \to 9^+ \Rightarrow \sqrt{x+7} \to 4^+ \Rightarrow 4-\sqrt{x+7} \to 0^- \)

(c) \[
\lim_{x \to \sqrt{2}} \frac{120 \text{arcsin}(x/2)}{x^2 + 4}
\]
\[
= \frac{120 \text{arcsin}(\frac{\sqrt{2}}{2})}{(\sqrt{2})^2 + 4}
\]
\[
= \frac{120 \cdot \frac{\pi}{4}}{6}
\]
\[
= \frac{5\pi}{1}
\]
(d) \[ \lim_{{x \to \infty}} \frac{13 + 5 \sin(9e^{3x} + 6)}{x^{10}} \]

\[ -1 \leq \sin(9e^{3x} + 6) \leq 1 \]

\[ -5 \leq 5 \sin(9e^{3x} + 6) \leq 5 \]

\[ 8 \leq 13 + 5 \sin(9e^{3x} + 6) \leq 18 \]

\[ \frac{8}{x^{10}} \leq \frac{13 + 5 \sin(9e^{3x} + 6)}{x^{10}} \leq \frac{18}{x^{10}} \quad \text{for } x \neq 0 \]

\[ \lim_{{x \to \infty}} \frac{8}{x^{10}} = 0 \quad \text{and} \quad \lim_{{x \to \infty}} \frac{18}{x^{10}} = 0 \]

By the squeeze theorem,

\[ \lim_{{x \to \infty}} \frac{13 + 5 \sin(9e^{3x} + 6)}{x^{10}} = 0 \]

2. (6 points each) Evaluate and simplify each of the following quantities.

(a) \[ \sin(2 \arctan(1.25)) = 2 \sin(\theta) \cos(\theta) \]

Let \( \theta = \arctan(1.25) \)

Then \( \tan(\theta) = 1.25 = \frac{5}{4} \)

for \( 0 < \theta < \frac{\pi}{2} \)

\[ \text{opp} = 5, \quad \text{adj} = 4 \]

pythagorean theorem

\[ \text{hyp} = \sqrt{41} \]

\[ \sin(\theta) = \frac{5}{\sqrt{41}} \]

\[ \cos(\theta) = \frac{4}{\sqrt{41}} \]

\[ 2 \sin(\theta) \cos(\theta) = 2 \cdot \frac{5}{\sqrt{41}} \cdot \frac{4}{\sqrt{41}} = \frac{40}{41} \]

(b) \[ e^{2 \ln(13)} - \ln(8e^9) + \ln(8) \]

\[ = e^{\ln(13^2)} - (\ln(8) + \ln(e^9)) + \ln(8) \]

\[ = e^{\ln(169)} - \ln(e^9) \]

\[ = 169 - 9 \]

\[ = 160 \]
3. (10 points) Let \( f(x) = 9 - 3x^2 \).

Use the definition of a derivative as a limit to prove that \( f'(x) = -6x \).

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

\[
\begin{align*}
   f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
   &= \lim_{h \to 0} \frac{9 - 3(x+h)^2 - (9 - 3x^2)}{h} \\
   &= \lim_{h \to 0} \frac{9 - 3(x^2 + 2xh + h^2) - 9 + 3x^2}{h} \\
   &= \lim_{h \to 0} \frac{9 - 3x^2 - 6xh - 3h^2 - 9 + 3x^2}{h} \\
   &= \lim_{h \to 0} \frac{-6xh - 3h^2}{h} \\
   &= \lim_{h \to 0} \frac{h(-6x - 3h)}{h} \\
   &= \lim_{h \to 0} (-6x - 3h) \\
   &= -6x
\end{align*}
\]
4. (10 points) Determine the $x$-intercept on the graph of $f(x) = e^{9x} - 121e^{7x}$. Simplify your answer.

\[
e^{9x} - 121e^{7x} = 0 \quad \Rightarrow \quad e^{2x} = 121
\]
\[
e^{7x}(e^{2x} - 121) = 0
\]
\[
e^{7x} > 0 \text{ for all } x
\]
\[
\text{Thus } e^{2x} - 121 = 0
\]
\[
x = \frac{1}{2} \ln(121)
\]
\[
x = \frac{1}{2} \ln(11^2) = \ln(11)
\]

5. (10 points) Write an equation for each horizontal asymptote on the graph of the following function. Use limits to justify your answer. We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

\[
\lim_{x \to \infty} \frac{56e^{-5x} - 30}{7e^{-5x} + 10} = \frac{56\cdot0 - 30}{7\cdot0 + 10} = -3
\]

Think about the graph of $y = e^{-5x}$

\[
\lim_{x \to -\infty} \frac{56e^{-5x} - 30}{7e^{-5x} + 10} = \infty
\]

(see graph above)

\[
= \lim_{x \to -\infty} \frac{56e^{-5x} - 30}{7e^{-5x} + 10} \cdot \frac{1/e^{-5x}}{1/e^{-5x}}
\]

\[
= \lim_{x \to -\infty} \frac{56 - 30e^{-5x}}{7 + 10e^{-5x}}
\]

\[
= \lim_{x \to -\infty} \frac{56 - 30e^{5x}}{7 + 10e^{5x}}
\]

\[
= \frac{56 - 30 \cdot 0}{7 + 10 \cdot 0} = 56/7 = 8
\]

horizontal asymptotes

$y = -3$ and $y = 8$
6. (10 points) The function \( w(x) = \frac{4}{5x^9 + 8} \) is one-to-one. Determine a formula for its inverse \( w^{-1}(x) \).

\[
\begin{align*}
 y &= \frac{4}{5x^9 + 8} \\
 x &= \frac{4}{5y^9 + 8} \\
 x(5y^9 + 8) &= 4 \\
 5xy^9 + 8x &= 4 \\
 5xy^9 &= 4 - 8x \\
 y^9 &= \frac{4 - 8x}{5x} \\
 y &= \sqrt[9]{\frac{4 - 8x}{5x}}
\end{align*}
\]

\[
 w^{-1}(x) = \sqrt[9]{\frac{4 - 8x}{5x}}
\]

7. (10 points) A function \( f(x) \) and its derivative \( f'(x) \) are given below. Determine the equation for the line which is tangent to the graph of \( f(x) \) at its y-intercept. Write your simplified answer in the form \( y = mx + b \).

\[
\begin{align*}
 f(x) &= \sin(8x) + \cos(6x) + 3 \\
 f'(x) &= 8 \cos(8x) - 6 \sin(6x)
\end{align*}
\]

Point: \( (0, f(0)) = (0, 4) \)

Slope: \( f'(0) = 8 \)

\[
y = 8x + 4
\]
8. (10 points) Use interval notation to state the domain of the given function.

\[ g(x) = \frac{\sqrt{x^2 - 64}}{\ln \left( \frac{6-x}{25} \right)} \]

From \( \sqrt{x^2 - 64} \), \[ x^2 - 64 \geq 0 \]
\[ x^2 \geq 64 \]
\[ \sqrt{x^2} \geq \sqrt{64} \]
\[ |x| \geq 8 \]
\[ x \leq -8 \text{ or } x \geq 8 \]

From \( \ln \left( \frac{6-x}{25} \right) \), \[ \frac{6-x}{25} > 0 \]
\[ 6-x > 0 \]
\[ 6 > x \]
\[ x < 6 \]

denominator equals 0 when \[ \ln \left( \frac{6-x}{25} \right) = 0 \]
\[ e^{\ln \left( \frac{6-x}{25} \right)} = e^0 \]
\[ \frac{6-x}{25} = 1 \]
\[ 6-x = 25 \]
\[ x = -19 \]
\[ \text{so } x \neq -19 \]

Domain of \( g(x) \): \((-\infty, -19) \cup (-19, 8]\)
Students – do not write on this page!

1a. (7 points) ______________________

1b. (7 points) ______________________

1c. (7 points) ______________________

1d. (7 points) ______________________

2a. (6 points) ______________________

2b. (6 points) ______________________

3. (10 points) ______________________

4. (10 points) ______________________

5. (10 points) ______________________

6. (10 points) ______________________

7. (10 points) ______________________

8. (10 points) ______________________

TOTAL (100 points) ________________