AL1 (MWF 10:00-10:50) AL3 (MWF 1:00-1:50) AL2 (MWF 3:00-3:50)

MATH 220 **Test 1** **Fall 2018**

Name Solutions

NetID ____________________

UIN ____________________

Circle your TA discussion section.

- ▷ **AD1**, TR 11:00-12:50, Adriana Morales
- ▷ **AD2**, TR 9:00-10:50, Hannah Burson
- ▷ **AD3**, TR 1:00-2:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Gayana Jayasinghe
- ▷ **ADB**, TR 9:00-9:50, Felix Clemen
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADE**, TR 12:00-12:50, Josh Wen
- ▷ **ADF**, TR 1:00-1:50, Nachiketa Adhikari
- ▷ **ADG**, TR 2:00-2:50, Lutian Zhao
- ▷ **ADH**, TR 3:00-3:50, Stathis Chrontsios
- ▷ **ADI**, TR 4:00-4:50, Stathis Chrontsios
- ▷ **ADJ**, TR 9:00-9:50, Gayana Jayasinghe
- ▷ **ADK**, TR 10:00-10:50, Madina Bolat
- ▷ **ADL**, TR 11:00-11:50, Chris Loa
- ▷ **ADM**, TR 12:00-12:50, Heeyeon Kim
- ▷ **ADN**, TR 1:00-1:50, Josh Wen
- ▷ **ADO**, TR 2:00-2:50, Kesav Krishnan
- ▷ **ADQ**, TR 10:00-10:50, Felix Clemen
- ▷ **ADR**, TR 9:00-9:50, Madina Bolat
- ▷ **ADS**, TR 12:00-12:50, Chris Loa
- ▷ **ADT**, TR 2:00-2:50, Nachiketa Adhikari
- ▷ **ADU**, TR 3:00-3:50, Kesav Krishnan
- ▷ **ADZ**, TR 9:00-9:50, Gidon Orelowitz

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																3	4	

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1. (7 points each) Evaluate the following limits and write your answers in simplified form. For infinite limits, you must clearly show whether the limit is ∞ or $-\infty$. We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

(a) $\lim_{x\to -2} \frac{x^2-3x-10}{x^7-16x^3}$

$$\frac{\to 0}{\to 0} = \lim_{x\to -2} \frac{(x-5)(x+2)}{x^3(x^4-16)}$$

$$= \lim_{x\to -2} \frac{(x-5)(x+2)}{x^3(x^2+4)(x^2-4)}$$

$$= \lim_{x\to -2} \frac{(x-5)(x+2)}{x^3(x^2+4)(x-2)(x+2)}$$

$$= \lim_{x\to -2} \frac{x-5}{x^3(x^2+4)(x-2)}$$

$$= \frac{-7}{256}$$

(b) $\lim_{x\to 9^+} \frac{14-x}{4-\sqrt{x+7}}$

$$\lim_{x\to 9^+} \frac{14-x}{4-\sqrt{x+7}} \begin{matrix} \to 5 \\ \to 0^- \end{matrix} = -\infty$$

note: $x \to 9^+ \Rightarrow \sqrt{x+7} \to 4^+$

$\Rightarrow 4-\sqrt{x+7} \to 0^-$

(c) $\lim_{x\to\sqrt{2}} \frac{120 \arcsin(x/2)}{x^2+4}$

$$= \frac{120 \arcsin\left(\frac{\sqrt{2}}{2}\right)}{(\sqrt{2})^2+4}$$

$$= \frac{120 \cdot \frac{\pi}{4}}{6}$$

$$= 5\pi$$

(d) $\lim\limits_{x\to\infty} \dfrac{13+5\sin\left(9e^{3x}+6\right)}{x^{10}}$

$$-1 \le \sin(9e^{3x}+6) \le 1$$
$$-5 \le 5\sin(9e^{3x}+6) \le 5$$
$$8 \le 13+5\sin(9e^{3x}+6) \le 18$$
$$\frac{8}{x^{10}} \le \frac{13+5\sin(9e^{3x}+6)}{x^{10}} \le \frac{18}{x^{10}} \text{ for } x \ne 0$$

$$\lim_{x\to\infty} \frac{8}{x^{10}} = 0 \text{ and } \lim_{x\to\infty} \frac{18}{x^{10}} = 0$$

By the squeeze theorem

$$\lim_{x\to\infty} \frac{13+5\sin(9e^{3x}+6)}{x^{10}} = 0$$

2. (6 points each) Evaluate and simplify each of the following quantities.

(a) $\sin\left(2\arctan(1.25)\right) = \sin(2\theta) = 2\sin(\theta)\cos(\theta)$

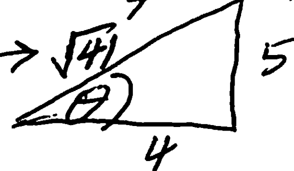
$$= 2 \cdot \frac{5}{\sqrt{41}} \cdot \frac{4}{\sqrt{41}}$$
$$= \frac{40}{41}$$

Let $\theta = \arctan(1.25)$

Then $\tan(\theta) = 1.25 = \frac{5}{4}$ $\left(\frac{\text{opp}}{\text{adj}}\right)$

for $0 < \theta < \frac{\pi}{2}$

pythagorean theorem



(b) $e^{2\ln(13)} - \ln\left(8e^{9}\right) + \ln(8)$

$$= e^{\ln(13^2)} - \left(\ln(8) + \ln(e^9)\right) + \ln(8)$$
$$= e^{\ln(169)} - \ln(e^9)$$
$$= 169 - 9$$
$$= 160$$

3. (10 points) Let $f(x) = 9 - 3x^2$.

Use the definition of a derivative as a limit to prove that $f'(x) = -6x$.

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{9 - 3(x+h)^2 - (9 - 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{9 - 3(x^2 + 2xh + h^2) - 9 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{9 - 3x^2 - 6xh - 3h^2 - 9 + 3x^2}{h}$$

$$= \lim_{h \to 0} \frac{-6xh - 3h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(-6x - 3h)}{h}$$

$$= \lim_{h \to 0} (-6x - 3h)$$

$$= -6x$$

4. (10 points) Determine the x-intercept on the graph of $f(x) = e^{9x} - 121e^{7x}$. Simplify your answer.

$$e^{9x} - 121e^{7x} = 0$$

$$e^{7x}(e^{2x} - 121) = 0$$

$e^{7x} > 0$ for all x

thus $e^{2x} - 121 = 0$

$$\Rightarrow e^{2x} = 121$$

$$\ln(e^{2x}) = \ln(121)$$

$$2x = \ln(121)$$

$$x = \tfrac{1}{2}\ln(121)$$

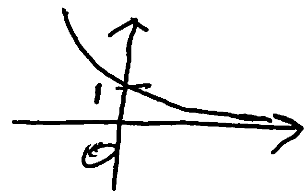
$$x = \tfrac{1}{2}\ln(11^2) = \ln(11)$$

5. (10 points) Write an equation for each horizontal asymptote on the graph of the following function. Use limits to justify your answer. We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

$$f(x) = \frac{56e^{-5x} - 30}{7e^{-5x} + 10}$$

$$\lim_{x\to\infty} \frac{56e^{-5x} - 30}{7e^{-5x} + 10} = \frac{56\cdot 0 - 30}{7\cdot 0 + 10} = -3$$

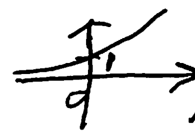
think about graph of $y = e^{-5x}$



$$\lim_{x\to-\infty} \frac{56e^{-5x} - 30 \to \infty}{7e^{-5x} + 10 \to \infty}$$ (see graph above)

$$= \lim_{x\to-\infty} \frac{56e^{-5x} - 30}{7e^{-5x} + 10} \cdot \frac{1/e^{-5x}}{1/e^{-5x}}$$

$$= \lim_{x\to-\infty} \frac{56 - 30/e^{-5x}}{7 + 10/e^{-5x}}$$

$$= \lim_{x\to-\infty} \frac{56 - 30e^{5x}}{7 + 10e^{5x}}$$ ($y = e^{5x}$ )

$$= \frac{56 - 30\cdot 0}{7 + 10\cdot 0} = \frac{56}{7} = 8$$

horizontal asymptotes

$y = -3$ and $y = 8$

6. (10 points) The function $w(x) = \dfrac{4}{5x^9 + 8}$ is one-to-one. Determine a formula for its inverse $w^{-1}(x)$.

$$y = \frac{4}{5x^9+8}$$

$$x = \frac{4}{5y^9+8}$$

$$x(5y^9+8) = 4$$

$$5xy^9+8x = 4$$

$$5xy^9 = 4-8x$$

$$y^9 = \frac{4-8x}{5x}$$

$$y = \sqrt[9]{\frac{4-8x}{5x}}$$

$$w^{-1}(x) = \sqrt[9]{\frac{4-8x}{5x}}$$

7. (10 points) A function $f(x)$ and its derivative $f'(x)$ are given below. Determine the equation for the line which is tangent to the graph of $f(x)$ at its y-intercept. Write your simplified answer in the form $y = mx + b$.

$$f(x) = \sin(8x) + \cos(6x) + 3$$

$$f'(x) = 8\cos(8x) - 6\sin(6x)$$

point: $(0, f(0)) = (0, 4)$

slope: $f'(0) = 8$

$$y = 8x + 4$$

8. (10 points) Use interval notation to state the domain of the given function.

$$g(x) = \frac{\sqrt{x^2 - 64}}{\ln\left(\frac{6-x}{25}\right)}$$

From $\sqrt{x^2-64}$, $x^2-64 \geq 0$

$x^2 \geq 64$

$\sqrt{x^2} \geq \sqrt{64}$

$|x| \geq 8$

$x \leq -8$ or $x \geq 8$

From $\ln\left(\frac{6-x}{25}\right)$, $\frac{6-x}{25} > 0$

$6-x > 0$

$6 > x$

$x < 6$

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denominator equals 0 when

$\ln\left(\frac{6-x}{25}\right) = 0$

$e^{\ln\left(\frac{6-x}{25}\right)} = e^0$

$\frac{6-x}{25} = 1$

$6-x = 25$

$x = -19$

so $x \neq -19$

Domain of $g(x)$

$(-\infty, -19) \cup (-19, -8]$

Students – do not write on this page!

1a. (7 points) ____________________

1b. (7 points) ____________________

1c. (7 points) ____________________

1d. (7 points) ____________________

2a. (6 points) ____________________

2b. (6 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

TOTAL (100 points) _______________