Math 220 (section AD?) **Quiz 9** **Fall 2018**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{(x^4-6)^2}{x^5}\,dx = \int \frac{(x^4)^2+2(x^4)(-6)+(-6)^2}{x^5}\,dx$

$= \int \frac{x^8-12x^4+36}{x^5}\,dx$

$= \int (x^3-12x^{-1}+36x^{-5})\,dx$

$= \frac{1}{4}x^4 - 12\ln|x| - 9x^{-4} + C$

(b) $\int \frac{\sec^2(x)-\tan^2(x)}{\sin^4(x)+\sin^2(x)\cos^2(x)}\,dx = \int \frac{\tan^2(x)+1-\tan^2(x)}{\sin^2(x)\left(\sin^2(x)+\cos^2(x)\right)}\,dx$

$= \int \frac{1}{\sin^2(x)\cdot 1}\,dx$

$= \int \csc^2(x)\,dx$

$= -\cot(x) + C$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_0^{1/2} \frac{3}{\sqrt{4-4x^2}}\,dx = \int_0^{1/2} \frac{3}{\sqrt{4(1-x^2)}}\,dx$$

$$= \int_0^{1/2} \frac{3}{2\sqrt{1-x^2}}\,dx$$

$$= \frac{3}{2}\int_0^{1/2} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{3}{2}\Big[\arcsin(x)\Big]_0^{1/2}$$

$$= \frac{3}{2}\Big[\arcsin(1/2) - \arcsin(0)\Big]$$

$$= \frac{3}{2}\Big[\frac{\pi}{6} - 0\Big]$$

$$= \frac{\pi}{4}$$

3. (2 points) At time t hours, a bacteria population is growing at a rate of $40t + 10$ bacteria per hour. If the population is 300 at time $t = 1$, then what is the population at time $t = 5$ hours?

(Pop. at $t=5$) = (Pop. at $t=1$) + (change in pop. bet. $t=1$ and $t=5$)

$$= 300 + \int_1^5 \left(\text{rate of change of pop}\right) dt$$

$$= 300 + \int_1^5 (40t+10)\,dt$$

$$= 300 + \Big[20t^2 + 10t\Big]_1^5$$

$$= 300 + \Big[20(5)^2 + 10(5)\Big] - \Big[20(1)^2 + 10(1)\Big]$$

$$= 300 + (500 + 50) - (20 + 10)$$

$$= 300 + 520$$

= 820 bacteria

4. (2 points) Suppose $g(x) = \int_3^{2x^3-24x+9} \left(t^8+5\right)^{42}\, dt$.

Determine each critical number of $g(x)$ and state whether the graph of $g(x)$ has a local maximum, a local minimum or neither at each of those x-values.

From part 1 of the Fundamental Theorem of Calculus along with the Chain Rule,

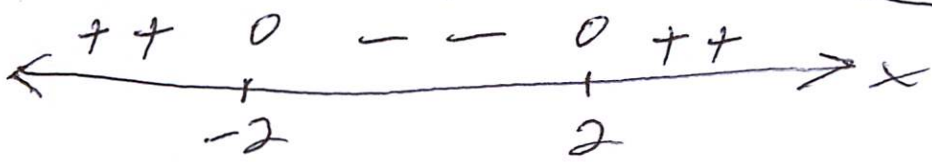
$$g'(x) = \left((2x^3-24x+9)^8+5\right)^{42} \cdot \frac{d}{dx}\left(2x^3-24x+9\right)$$

$$= \left((2x^3-24x+9)^8+5\right)^{42} \cdot (6x^2-24)$$

$$= \underbrace{\left((2x^3-24x+9)^8+5\right)^{42}}_{\text{always positive}} \cdot 6(x+2)(x-2)$$

$\boxed{\begin{array}{l}\text{critical numbers} \\ \hline x=-2 \\ x=2\end{array}}$

values of $g'(x)$



using the first derivative test,

g has a local max. when $x=-2$
and
g has a local min. when $x=2$