Math 220 **Quiz 8 (take-home)** **Fall 2018**

Name Solutions

(circle your TA discussion section)

- **AD1**, TR 11:00-12:50, Adriana Morales
- **AD2**, TR 9:00-10:50, Hannah Burson
- **AD3**, TR 1:00-2:50, Dana Neidinger
- **ADA**, TR 8:00-8:50, Gayana Jayasinghe
- **ADB**, TR 9:00-9:50, Felix Clemen
- **ADC**, TR 10:00-10:50, Lutian Zhao
- **ADD**, TR 11:00-11:50, Gidon Orelowitz
- **ADE**, TR 12:00-12:50, Josh Wen
- **ADF**, TR 1:00-1:50, Nachiketa Adhikari
- **ADG**, TR 2:00-2:50, Lutian Zhao
- **ADH**, TR 3:00-3:50, Stathis Chrontsios
- **ADI**, TR 4:00-4:50, Stathis Chrontsios
- **ADJ**, TR 9:00-9:50, Gayana Jayasinghe
- **ADK**, TR 10:00-10:50, Madina Bolat
- **ADL**, TR 11:00-11:50, Chris Loa
- **ADM**, TR 12:00-12:50, Heeyeon Kim
- **ADN**, TR 1:00-1:50, Josh Wen
- **ADO**, TR 2:00-2:50, Kesav Krishnan
- **ADQ**, TR 10:00-10:50, Felix Clemen
- **ADR**, TR 9:00-9:50, Madina Bolat
- **ADS**, TR 12:00-12:50, Chris Loa
- **ADT**, TR 2:00-2:50, Nachiketa Adhikari
- **ADU**, TR 3:00-3:50, Kesav Krishnan
- **ADZ**, TR 9:00-9:50, Gidon Orelowitz

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official discussion period on Tuesday, November 6th.**
- **TAs and Tutors – Do not help students with these specific problems until all discussion sections have turned in the quiz.**

1. (2 points) Find a formula for $f(x)$ given that $f''(x) = 5e^x + 2\sin(x) + 6$, $f(0) = 9$ and $f'(0) = -11$.

$f''(x) = 5e^x + 2\sin(x) + 6 \Rightarrow f'(x) = 5e^x - 2\cos(x) + 6x + C_1$

$f'(0) = -11 \Rightarrow 5e^0 - 2\cos(0) + 6(0) + C_1 = -11 \Rightarrow C_1 = -14$

$f'(x) = 5e^x - 2\cos(x) + 6x - 14 \Rightarrow f(x) = 5e^x - 2\sin(x) + 3x^2 - 14x + C_2$

$f(0) = 9 \Rightarrow 5e^0 - 2\sin(0) + 3(0)^2 - 14(0) + C_2 = 9 \Rightarrow C_2 = 4$

$f(x) = 5e^x - 2\sin(x) + 3x^2 - 14x + 4$

2. (2 points) Evaluate the following limit. Use proper notation in your evaluation of this limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{6nk - 12k^2}{n^3} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{6nk}{n^3} - \frac{12k^2}{n^3} \right)$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{6nk}{n^3} - \sum_{k=1}^{n} \frac{12k^2}{n^3} \right)$$

$$= \lim_{n\to\infty} \left(\frac{6}{n^2} \sum_{k=1}^{n} k - \frac{12}{n^3} \sum_{k=1}^{n} k^2 \right)$$

$$= \lim_{n\to\infty} \left(\frac{6}{n^2} \cdot \frac{n(n+1)}{2} - \frac{12}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right)$$

$$= \frac{6}{2} - \frac{12 \cdot 2}{6}$$

$$= -1$$

3. (2 points) From section 5.2 we have the following property of definite integrals.

- If $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

Use this property to carefully explain why the following inequality holds.

$$0.75 \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq 3$$

$$-1 \leq \sin(4e^x) \leq 1$$

$$-3 \leq 3\sin(4e^x) \leq 3$$

$$2 \leq 5+3\sin(4e^x) \leq 8$$

$$\frac{2}{x^3} \leq \frac{5+3\sin(4e^x)}{x^3} \leq \frac{8}{x^3} \quad \text{for } x>0$$

$$\int_1^2 \frac{2}{x^3}\,dx \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq \int_1^2 \frac{8}{x^3}\,dx$$

$$\int_1^2 2x^{-3}\,dx \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq \int_1^2 8x^{-3}\,dx$$

$$-x^{-2}\Big|_1^2 \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq -4x^{-2}\Big|_1^2$$

$$-\frac{1}{x^2}\Big|_1^2 \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq \frac{-4}{x^2}\Big|_1^2$$

$$\frac{-1}{2^2}-\left(\frac{-1}{1^2}\right) \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq \frac{-4}{2^2}-\left(\frac{-4}{1^2}\right)$$

$$0.75 \leq \int_1^2 \frac{5+3\sin(4e^x)}{x^3}\,dx \leq 3$$

4. (2 points) Express the definite integral as the limit of Riemann sums. Do not evaluate the limit.

$$\int_{-3}^{5} x^2 e^{\sin(x)}\,dx = \lim_{n\to\infty} \sum_{K=1}^{n} f(x_K^*)\Delta x$$

$$= \lim_{n\to\infty} \sum_{K=1}^{n} (x_K^*)^2 e^{\sin(x_K^*)} \Delta x$$

$$= \lim_{n\to\infty} \sum_{K=1}^{n} \left(-3+K\cdot\frac{8}{n}\right)^2 e^{\sin\left(-3+K\cdot\frac{8}{n}\right)} \cdot \frac{8}{n}$$

$f(x) = x^2 e^{\sin(x)}$

$\Delta x = \frac{b-a}{n}$

$= \frac{5-(-3)}{n} = \frac{8}{n}$

$x_K^* = a + K\Delta x$

$= -3 + K\cdot\frac{8}{n}$

Using $\sum_{K=1}^{n}$, I chose a right Riemann sum.

For a left sum, replace K with $K-1$.

For a midpoint sum, replace K with $K-\frac{1}{2}$.

5. (2 points) Fill in the missing information from the second sum so that the equality holds. Do not evaluate the sum.

$$\sum_{k=5}^{n-1} k\ln(k) = \sum_{j=1}^{n-5} (j+4)\ln(j+4)$$

$$\sum_{K=5}^{n-1} K\ln(K) = 5\ln(5) + 6\ln(6) + \cdots + (n-1)\ln(n-1) \quad ✓$$

$$\sum_{j=1}^{n-5} (j+4)\ln(j+4) = 5\ln(5) + 6\ln(6) + \cdots + (n-1)\ln(n-1) \quad ✓$$