Math 220 **Quiz 7 (take-home)** **Fall 2018**

Name Solutions

(circle your TA discussion section)

- ▹ **AD1**, TR 11:00-12:50, Adriana Morales
- ▹ **AD2**, TR 9:00-10:50, Hannah Burson
- ▹ **AD3**, TR 1:00-2:50, Dana Neidinger
- ▹ **ADA**, TR 8:00-8:50, Gayana Jayasinghe
- ▹ **ADB**, TR 9:00-9:50, Felix Clemen
- ▹ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▹ **ADD**, TR 11:00-11:50, Gidon Orelowitz
- ▹ **ADE**, TR 12:00-12:50, Josh Wen
- ▹ **ADF**, TR 1:00-1:50, Nachiketa Adhikari
- ▹ **ADG**, TR 2:00-2:50, Lutian Zhao
- ▹ **ADH**, TR 3:00-3:50, Stathis Chrontsios
- ▹ **ADI**, TR 4:00-4:50, Stathis Chrontsios
- ▹ **ADJ**, TR 9:00-9:50, Gayana Jayasinghe
- ▹ **ADK**, TR 10:00-10:50, Madina Bolat
- ▹ **ADL**, TR 11:00-11:50, Chris Loa
- ▹ **ADM**, TR 12:00-12:50, Heeyeon Kim
- ▹ **ADN**, TR 1:00-1:50, Josh Wen
- ▹ **ADO**, TR 2:00-2:50, Kesav Krishnan
- ▹ **ADQ**, TR 10:00-10:50, Felix Clemen
- ▹ **ADR**, TR 9:00-9:50, Madina Bolat
- ▹ **ADS**, TR 12:00-12:50, Chris Loa
- ▹ **ADT**, TR 2:00-2:50, Nachiketa Adhikari
- ▹ **ADU**, TR 3:00-3:50, Kesav Krishnan
- ▹ **ADZ**, TR 9:00-9:50, Gidon Orelowitz

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- You are not allowed to use a calculator, Wolfram Alpha, or any similar technology.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Friday, October 19th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (10am, 1pm, 3pm).**

1. (3 points) A function $f(x)$ is differentiable on the interval $(-\infty, \infty)$. Its first derivative is given below. Find the intervals of concavity and the x-value for each inflection point of $f(x)$.

$f'(x) = e^{2x}\left(4x^3 - 18x^2 + 18x - 9\right)$

$$f''(x) = \frac{d}{dx}(e^{2x})\cdot(4x^3-18x^2+18x-9) + e^{2x}\cdot\frac{d}{dx}(4x^3-18x^2+18x-9)$$

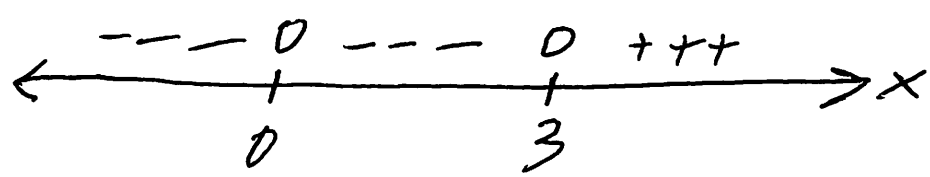
$$= 2e^{2x}(4x^3-18x^2+18x-9) + e^{2x}(12x^2-36x+18)$$

$$= 8x^3e^{2x} - 36x^2e^{2x} + 36xe^{2x} - 18e^{2x} + 12x^2e^{2x} - 36xe^{2x} + 18e^{2x}$$

$$= 8x^3e^{2x} - 24x^2e^{2x}$$

$$= 8x^2e^{2x}(x-3)$$

values of $f''(x)$



f is concave down on $(-\infty, 0)$
f is concave down on $(0, 3)$
f is concave up on $(3, \infty)$

okay to say f is concave down on $(-\infty, 3)$

There is an inflection point at $x = 3$, (it switches concavity there)

2. (3 points) Evaluate the following limit.

$$\lim_{x\to 0} (\cos(4x))^{1/x^2}$$

(indeterminate form; 1^{∞})

$$\lim_{x\to 0} (\cos(4x))^{1/x^2} = \lim_{x\to 0} e^{\ln\left((\cos(4x))^{1/x^2}\right)}$$

(Just using $w = e^{\ln w}$)

$$= \lim_{x\to 0} e^{\frac{1}{x^2}\cdot \ln(\cos(4x))}$$

$$= \lim_{x\to 0} e^{\frac{\ln(\cos(4x))}{x^2}}$$

$$= e^{\lim_{x\to 0} \frac{\ln(\cos(4x)) \to 0}{x^2 \to 0}}$$

$$\overset{H}{=} e^{\lim_{x\to 0} \frac{\frac{1}{\cos(4x)}\cdot -\sin(4x)\cdot 4}{2x}}$$

$$= e^{\lim_{x\to 0} \frac{-2\sin(4x) \to 0}{x\cos(4x) \to 0}}$$

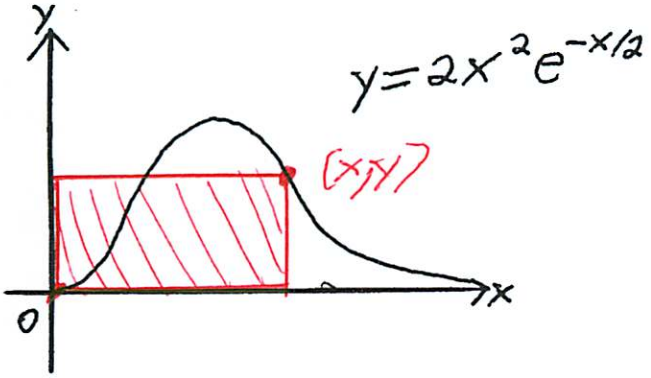
$$\overset{H}{=} e^{\lim_{x\to 0} \frac{-8\cos(4x)}{1\cdot\cos(4x) + x(-4\sin(4x))}}$$

$$= e^{-8}$$

$\overset{H}{=}$ means "by l'Hospital's Rule"

3. (4 points) Determine the largest possible area for a rectangle which satisfies all three of the following conditions.

- The rectangle's bottom edge lies on the x-axis.
- The rectangle's bottom left corner is the point $(0, 0)$.
- The rectangle's top right corner is the point (x, y) on the curve $y = 2x^2e^{-x/2}$ with $x > 0$.



Area = width·height

$A = x \cdot y$

$A = x \cdot 2x^2 e^{-x/2}$

$A = 2x^3 e^{-x/2}$

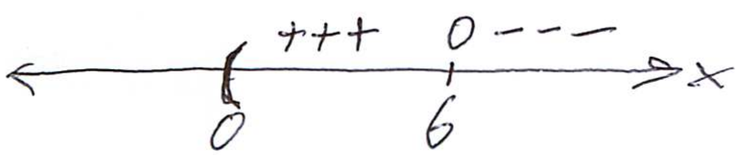
minimize A on $(0,\infty)$

$A' = \frac{d}{dx}(2x^3) \cdot e^{-x/2} + 2x^3 \cdot \frac{d}{dx}(e^{-x/2})$

$A' = 6x^2 e^{-x/2} + 2x^3 \cdot -\frac{1}{2} e^{-x/2}$

$A' = x^2 e^{-x/2}(6 - x)$

values of A' for $x > 0$



The 1st derivative test only shows a local max at $x=6$, but the sign chart shows it is also the absolute max.

The area for $x=6$ is

$A = 2(6)^3 e^{-6/2}$

$= 432e^{-3}$

$= \frac{432}{e^3}$