Math 220 (section AD?) Quiz 6 Fall 2018

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Suppose that A represents the number of grams of a radioactive substance at time t seconds. Given that $\frac{dA}{dt} = -0.2A$, how long does it take 20 grams of this substance to be reduced to 4 grams?

$\frac{dA}{dt} = -0.2A \Rightarrow \boxed{A = Ce^{-0.2t}}$

At $t=0$, $A = 20$ grams

Thus, $20 = Ce^{-0.2(0)} = C$

$\boxed{A = 20e^{-0.2t}}$

Set $A = 4$ grams

$4 = 20e^{-0.2t}$

$e^{-0.2t} = \frac{4}{20} = \frac{1}{5}$

$\ln(e^{-0.2t}) = \ln(\frac{1}{5})$

$-0.2t = \ln(1/5)$

$t = \frac{\ln(1/5)}{-0.2} = \frac{\ln(1) - \ln(5)}{-0.2}$

$t = 5\ln(5)$ seconds

2. (4 points) The height of a remote-controlled drone in feet above ground for $t \geq 0$ seconds is given by the following function.

$$h(t) = \frac{t^{10}}{3e^{2t}}$$

What is the maximum height obtained by the drone?

$$h'(t) = \frac{10t^9 \cdot 3e^{2t} - t^{10} \cdot 3e^{2t} \cdot 2}{(3e^{2t})^2}$$

$$= \frac{3t^9 e^{2t}(10 - 2t)}{(3e^{2t})^2}$$

$$= \frac{t^9(10-2t)}{3e^{2t}}$$

values of $h'(t)$ for $t \geq 0$

0 $+++$ 0 $---$ $\rightarrow t$

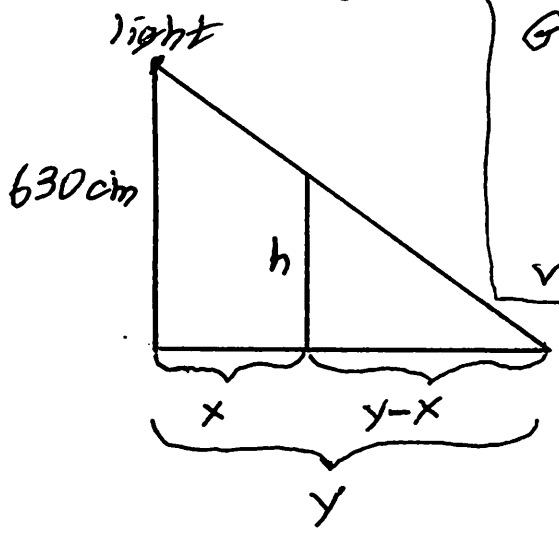
0 $\quad$ 5

h increasing $\quad$ h decreasing

The maximum height is obtained when $t = 5$ seconds

$$h(5) = \frac{5^{10}}{3e^{2 \cdot 5}} = \boxed{\frac{5^{10}}{3e^{10}} \text{ ft}}$$

3. (3 points) A street light is mounted at the top of a 630 *cm* pole. As a woman walks away from the pole, the tip of her shadow is moving 40% faster than she is moving. What is the woman's height?



Given: $\frac{dy}{dt} = \frac{dx}{dt} + 0.4\frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = 1.4\frac{dx}{dt}$

Want: h (the woman's height)

From similar triangles,

$$\frac{630}{h} = \frac{y}{y-x} \Rightarrow hy = 630y - 630x$$

$$\frac{d}{dt}(hy) = \frac{d}{dt}(630y - 630x)$$

$$h\frac{dy}{dt} = 630\frac{dy}{dt} - 630\frac{dx}{dt}$$

$$h = \frac{630\frac{dy}{dt} - 630\frac{dx}{dt}}{\frac{dy}{dt}}$$

$$h = \frac{630(1.4\frac{dx}{dt}) - 630\frac{dx}{dt}}{1.4\frac{dx}{dt}}$$

$$= \frac{630\frac{dx}{dt}(1.4 - 1)}{1.4\frac{dx}{dt}}$$

$$= \frac{630(0.4)}{1.4} = 630 \cdot \frac{2}{7} = 180 \text{ cm}$$