Math 220 (section AD?) **Quiz 5** **Fall 2018**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Compute $f'(x)$ given that $f(x) = \arctan\left(\sqrt{x^2+3}\right)$.

$$f(x) = \arctan\left((x^2+3)^{1/2}\right)$$

$$f'(x) = \frac{1}{\left((x^2+3)^{1/2}\right)^2+1} \cdot \frac{d}{dx}\left((x^2+3)^{1/2}\right)$$

$$f'(x) = \frac{1}{x^2+3+1} \cdot \frac{1}{2}(x^2+3)^{-1/2} \cdot \frac{d}{dx}(x^2+3)$$

$$f'(x) = \frac{1}{x^2+4} \cdot \frac{1}{2\sqrt{x^2+3}} \cdot 2x$$

$$f'(x) = \frac{x}{(x^2+4)\sqrt{x^2+3}}$$

2. (3 points) Given $w(x) = \ln\left(5\sin(x) + x^3 + 9\right)$, find its second derivative $w''(x)$.

$$w'(x) = \frac{1}{5\sin(x)+x^3+9} \cdot \frac{d}{dx}\left(5\sin(x)+x^3+9\right)$$

$$w'(x) = \frac{1}{5\sin(x)+x^3+9} \cdot \left(5\cos(x)+3x^2\right)$$

$$w'(x) = \frac{5\cos(x)+3x^2}{5\sin(x)+x^3+9}$$

$$w''(x) = \frac{\frac{d}{dx}\left(5\cos(x)+3x^2\right)\left(5\sin(x)+x^3+9\right) - \left(5\cos(x)+3x^2\right)\cdot\frac{d}{dx}\left(5\sin(x)+x^3+9\right)}{\left(5\sin(x)+x^3+9\right)^2}$$

$$w''(x) = \frac{\left(-5\sin(x)+6x\right)\left(5\sin(x)+x^3+9\right) - \left(5\cos(x)+3x^2\right)\left(5\cos(x)+3x^2\right)}{\left(5\sin(x)+x^3+9\right)^2}$$

3. (3 points) Find the equation of the line tangent to the given curve at the point $(-1, 2)$.

$$3x^2y + 2 = \left(2x + y^2\right)^3$$

$$\frac{d}{dx}(3x^2y+2) = \frac{d}{dx}\left((2x+y^2)^3\right)$$

$$\frac{d}{dx}(3x^2)\cdot y + 3x^2 \frac{d}{dx}(y) + 0 = 3(2x+y^2)^2 \cdot \frac{d}{dx}(2x+y^2)$$

$$6xy + 3x^2\frac{dy}{dx} = 3(2x+y^2)^2 \cdot \left(2 + 2y\frac{dy}{dx}\right)$$

$$6xy + 3x^2\frac{dy}{dx} = 6(2x+y^2)^2 + 6y(2x+y^2)^2\frac{dy}{dx}$$

$$3x^2\frac{dy}{dx} - 6y(2x+y^2)^2\frac{dy}{dx} = 6(2x+y^2)^2 - 6xy$$

$$\frac{dy}{dx}\left(3x^2 - 6y(2x+y^2)^2\right) = 6(2x+y^2)^2 - 6xy$$

$$\frac{dy}{dx} = \frac{6(2x+y^2)^2 - 6xy}{3x^2 - 6y(2x+y^2)^2}$$

at $(-1,2)$, $\dfrac{dy}{dx} = \dfrac{6(2(-1)+(2)^2)^2 - 6(-1)(2)}{3(-1)^2 - 6(2)(2(-1)+(2)^2)^2}$

$$= \frac{6\cdot 4 + 12}{3 - 12(4)} = \frac{36}{-45} = -\frac{4}{5}$$

Point: $(-1, 2)$ slope: $-\frac{4}{5}$

tangent line: $y - 2 = -\frac{4}{5}(x - (-1))$

$$y = -\frac{4}{5}x + \frac{6}{5}$$

4. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

$$y = \left(x^2+4\right)^{x^3}$$

metod 1

$$\ln(y) = \ln\left((x^2+4)^{x^3}\right)$$

$$\ln(y) = x^3 \cdot \ln(x^2+4)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x^3 \cdot \ln(x^2+4))$$

$$\frac{1}{y}\frac{dy}{dx} = 3x^2 \cdot \ln(x^2+4) + x^3 \cdot \frac{1}{x^2+4} \cdot 2x$$

$$\frac{dy}{dx} = y\left(3x^2\ln(x^2+4) + \frac{2x^4}{x^2+4}\right)$$

$$\frac{dy}{dx} = (x^2+4)^{x^3}\left(3x^2\ln(x^2+4) + \frac{2x^4}{x^2+4}\right)$$

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method 2

$$y = (x^2+4)^{x^3}$$

$$y = e^{\ln((x^2+4)^{x^3})}$$

$$y = e^{x^3 \cdot \ln(x^2+4)}$$

$$\frac{dy}{dx} = e^{x^3\ln(x^2+4)} \cdot \frac{d}{dx}(x^3 \cdot \ln(x^2+4))$$

$$\frac{dy}{dx} = e^{x^3 \cdot \ln(x^2+4)} \cdot \left(3x^2 \cdot \ln(x^2+4) + x^3 \cdot \frac{1}{x^2+4} \cdot 2x\right)$$

$$\frac{dy}{dx} = (x^2+4)^{x^3}\left(3x^2\ln(x^2+4) + \frac{2x^4}{x^2+4}\right)$$