Math 220 (section AD?) **Quiz 4** **Fall 2018**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) The normal line to a curve at a point P is the line through P that is perpendicular to the tangent line at P. Find the slope of the normal line to the given curve at $x = \pi/6$. Simplify your answer.

$$f(x) = 12\tan(x) + 3\sec(x)$$

$$f'(x) = 12\sec^2(x) + 3\sec(x)\tan(x)$$

$$f'(\tfrac{\pi}{6}) = 12\sec^2(\tfrac{\pi}{6}) + 3\sec(\tfrac{\pi}{6})\tan(\tfrac{\pi}{6})$$

$$= \frac{12}{\cos^2(\frac{\pi}{6})} + \frac{3}{\cos(\frac{\pi}{6})} \cdot \frac{\sin(\pi/6)}{\cos(\pi/6)}$$

$$= \frac{12}{(\frac{\sqrt{3}}{2})^2} + \frac{3}{(\frac{\sqrt{3}}{2})} \cdot \frac{1/2}{(\sqrt{3}/2)}$$

$$= 12 \cdot \frac{4}{3} + \frac{3}{2} \cdot \frac{4}{3}$$

$$= 18$$

slope of tangent line is 18

slope of normal line is $-\frac{1}{18}$

2. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $\theta = x^3 \left(\frac{x\sqrt{x}}{\sqrt[3]{x}}\right)^6$ (simplify your answer)

$$\theta = x^3\left(\frac{x^{3/2}}{x^{1/3}}\right)^6$$

$$\theta = x^3\left(\frac{x^{\frac{3}{2}\cdot 6}}{x^{\frac{1}{3}\cdot 6}}\right)$$

$$\theta = x^3\left(\frac{x^9}{x^2}\right)$$

$$\theta = x^{10}$$

$$\frac{d\theta}{dx} = 10x^9$$

(b) $q = \frac{1}{t^6} + \ln\left(5e^2\right)$

$$q = t^{-6} + \text{constant}$$

$$\frac{dq}{dt} = -6t^{-7}$$

(c) $r = \frac{\cot(w)}{\sqrt{w} + \sin(w)}$

$$\frac{dr}{dw} = \frac{\frac{d}{dw}(\cot(w))\cdot(\sqrt{w}+\sin(w)) - \cot(w)\cdot\frac{d}{dw}(\sqrt{w}+\sin(w))}{(\sqrt{w}+\sin(w))^2}$$

$$\frac{dr}{dw} = \frac{-\csc^2(w)(\sqrt{w}+\sin(w)) - \cot(w)\left(\frac{1}{2}w^{-1/2}+\cos(w)\right)}{(\sqrt{w}+\sin(w))^2}$$

3. (2 points) Determine the x-value for each point on the curve where the tangent line is horizontal.

$$g(x) = e^x\left(x^2 + x - 11\right)$$

$$g'(x) = \frac{d}{dx}(e^x) \cdot (x^2 + x - 11) + e^x \cdot \frac{d}{dx}(x^2 + x - 11)$$

$$g'(x) = e^x(x^2 + x - 11) + e^x(2x + 1)$$

$$g'(x) = e^x(x^2 + x - 11 + 2x + 1)$$

$$g'(x) = e^x(x^2 + 3x - 10)$$

$$g'(x) = e^x(x+5)(x-2)$$

horizontal lines have slope 0

$$0 = e^x(x+5)(x-2)$$

never 0

$x+5=0$ or $x-2=0$

$x=-5$ or $x=2$