Math 220 (section AD?) Quiz 2 Fall 2018

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (1 point) Simplify the following quantity.

$15e^{2\ln(\sqrt{3}/3)}$

$= 15e^{\ln\left(\left(\frac{\sqrt{3}}{3}\right)^2\right)}$

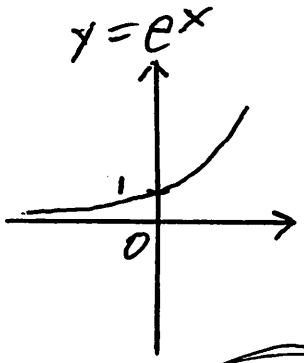
$= 15\left(\frac{\sqrt{3}}{3}\right)^2$

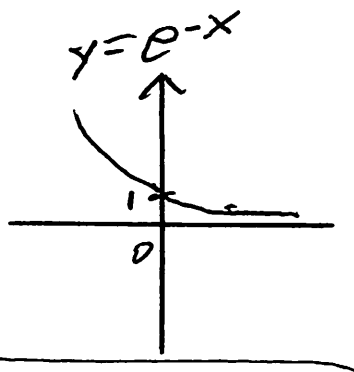
$= 15\left(\frac{3}{9}\right)$

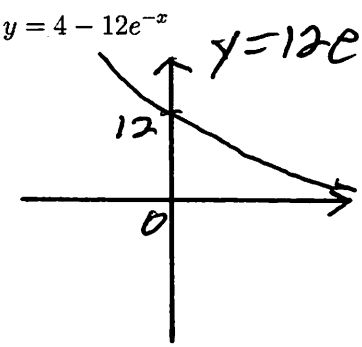
$= 15\left(\frac{1}{3}\right)$

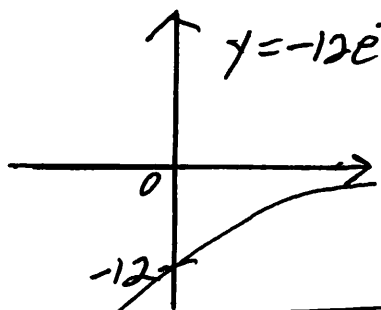
$= 5$

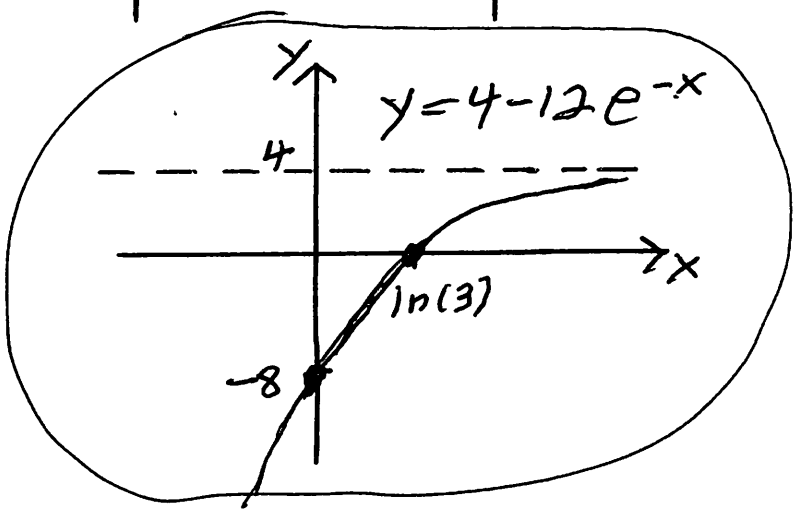
2. (3 points) Carefully sketch a graph of the following function. You should clearly label the value of any x-intercepts, y-intercepts, or asymptotes.











no vertical asymptote

horizontal asymptote: $y=4$

y-intercept: $y=4-12e^{-(0)}$

$y=-8$

x-intercept: $0=4-12e^{-x}$

$12e^{-x}=4$

$e^{-x}=\frac{4}{12}=\frac{1}{3}$

$\frac{1}{e^x}=\frac{1}{3}$

$e^x=3 \Rightarrow x=\ln(3)$

3. (3 points) Determine all values of x which satisfy the equation below.

$$\ln(x+5)+\ln(x+7)=\ln(3)$$

$\ln((x+5)(x+7)) = \ln(3)$

$e^{\ln((x+5)(x+7))} = e^{\ln(3)}$

$(x+5)(x+7) = 3$

$x^2+12x+35 = 3$

$x^2+12x+32 = 0$

$(x+8)(x+4) = 0$

This last equation has solutions -8 and -4 but -8 is not in the domain of original equation.

only solution: $x = -4$

4. (3 points) The function $w(x) = \dfrac{4}{5+3e^{2x}}$ is one-to-one on its domain. Determine a formula for its inverse $w^{-1}(x)$.

$y = \dfrac{4}{5+3e^{2x}}$

$x = \dfrac{4}{5+3e^{2y}}$

$x(5+3e^{2y}) = 4$

$5x+3xe^{2y} = 4$

$3xe^{2y} = 4-5x$

$e^{2y} = \dfrac{4-5x}{3x}$

$\ln(e^{2y}) = \ln\left(\dfrac{4-5x}{3x}\right)$

$2y = \ln\left(\dfrac{4-5x}{3x}\right)$

$y = \dfrac{1}{2}\ln\left(\dfrac{4-5x}{3x}\right)$

$w^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{4-5x}{3x}\right)$