(circle your TA discussion section)

- AD1, TR 11:00-12:50, Adriana Morales
- AD2, TR 9:00-10:50, Hannah Burson
- AD3, TR 1:00-2:50, Dana Neidinger
- ADA, TR 8:00-8:50, Gayana Jayasinghe
- ADB, TR 9:00-9:50, Felix Clemen
- ADC, TR 10:00-10:50, Lutian Zhao
- ADD, TR 11:00-11:50, Gidon Orelowitz
- ADE, TR 12:00-12:50, Josh Wen
- ADF, TR 1:00-1:50, Nachiketa Adhikari
- ADG, TR 2:00-2:50, Lutian Zhao
- ADH, TR 3:00-3:50, Stathis Chrontsios
- ADI, TR 4:00-4:50, Stathis Chrontsios
- ADJ, TR 9:00-9:50, Gayana Jayasinghe
- ADK, TR 10:00-10:50, Madina Bolat
- ADL, TR 11:00-11:50, Chris Loa
- ADM, TR 12:00-12:50, Heeyeon Kim
- ADN, TR 1:00-1:50, Josh Wen
- ADO, TR 2:00-2:50, Kessav Krishnan
- ADQ, TR 10:00-10:50, Felix Clemen
- ADR, TR 9:00-9:50, Madina Bolat
- ADS, TR 12:00-12:50, Chris Loa
- ADT, TR 2:00-2:50, Nachiketa Adhikari
- ADU, TR 3:00-3:50, Kessav Krishnan
- ADZ, TR 9:00-9:50, Gidon Orelowitz

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- The only computational technology allowed is a calculator for basic arithmetic (+, -, ×, ÷).
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- The quiz is due at the beginning of your lecture period on Friday, November 30th.
- TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (10am, 1pm, 3pm).
1. (3 points) A calculator gives an estimate of 4.979878868 for the value of \( \sqrt[4]{615} \).

Using the techniques of linear approximation found in section 3.10, show that it is possible to obtain a very similar estimate of 4.98 without the use of any technology.

\[
f(x) = \sqrt[4]{x} = x^{1/4} \quad \Rightarrow \quad f'(x) = \frac{1}{4}x^{-3/4}
\]

Tangent line at \( x=625 \)

Point: \( (625, \sqrt[4]{625}) = (625, 5) \)

Slope: \( f'(625) = \frac{1}{4}(625)^{-3/4} = \frac{1}{4(625^{1/4})^3} = \frac{1}{45} = \frac{1}{500} \)

\[
L(x) = 5 + \frac{1}{500}(x-625)
\]

\[
f(x) \approx L(x) \text{ near point of tangency}
\]

\[
\sqrt[4]{615} \approx 5 + \frac{1}{500}(615-625)
\]

\[
\sqrt[4]{615} \approx 5 - \frac{10}{500} = 5 - 0.02 = 4.98
\]

\[\sqrt[4]{615} \approx 4.98 \text{ (seen as overestimate from graphs)}\]
2. (3 points) Suppose that $f(x)$ and $g(x)$ are differentiable everywhere and satisfy the following conditions.

- $f(x)$ is an odd function
- $g(x)$ is an even function
- $f(-2) = 8$
- $g(-2) = -3$
- $f'(2) = 1/6$
- $g'(2) = 1/4$

Let $w(x) = f(x)g(x)$. Use the properties of even and odd functions along with the techniques of linear approximation found in section 3.10 to estimate the value of $w(2.2)$. Simplify your answer.

\[ f \text{ is odd and } f(-2) = 8 \implies f(2) = -8 \]
\[ g \text{ is even and } g(-2) = -3 \implies g(2) = -3 \]
\[ w(x) = f(x)g(x) \implies w'(x) = f'(x)g(x) + f(x)g'(x) \]
\[ w(2) = f(2)g(2) = (-8)(-3) = 24 \]
\[ w'(2) = f'(2)g(2) + f(2)g'(2) = \left(\frac{1}{6}\right)(-3) + (-8)(\frac{1}{4}) = -\frac{5}{2} \]

Point: $(2, 24)$
Slope: $-\frac{5}{2}$
Tangent line: $y = 24 - \frac{5}{2}(x - 2)$

Linear approximation using tangent line to $w(x)$ at $x = 2$.

\[ w(x) \approx 24 - \frac{5}{2}(x - 2) \text{ for } x \text{ near } 2 \]
\[ w(2.2) \approx 24 - \frac{5}{2}(2.2 - 2) \]
\[ w(2.2) \approx 23.5 \]
Another estimate using linear approximation

\[ f \text{ is odd and } f(-2) = -8 \Rightarrow f(2) = -8 \text{ (point: (2, -8))} \]
\[ f'(2) = \frac{1}{6} \text{ (slope: } \frac{1}{6}) \]
\[ y - (-8) = \frac{1}{6} (x - 2) \]
\[ y = -8 + \frac{1}{6} (x - 2) \]
\[ f(x) \approx -8 + \frac{1}{6} (x - 2) \text{ for } x \text{ near 2} \]
\[ f(2.2) \approx -8 + \frac{1}{6} (2.2 - 2) \]
\[ f(2.2) \approx -8 + \frac{1}{6} \cdot 0.2 = -7.9667 \]

\[ g \text{ is even and } g(-2) = -3 \Rightarrow g(2) = -3 \text{ (point: (2, -3))} \]
\[ g'(2) = \frac{1}{4} \text{ (slope: } \frac{1}{4}) \]
\[ y - (-3) = \frac{1}{4} (x - 2) \]
\[ y = -3 + \frac{1}{4} (x - 2) \]
\[ g(x) \approx -3 + \frac{1}{4} (x - 2) \text{ for } x \text{ near 2} \]
\[ g(2.2) \approx -3 + \frac{1}{4} (2.2 - 2) \]
\[ g(2.2) \approx -3 + \frac{1}{4} \cdot 0.2 = -2.95 \]

\[ w(x) = f(x)g(x) \]
\[ w(2.2) = f(2.2)g(2.2) = \left( -\frac{239}{30} \right) \left( -\frac{59}{80} \right) \approx 23.50167 \]
3. (4 points) There is one value of \( x \) for which the \( y \)-value on the graph of \( g(x) = 2x^3 + 10x + 9 \) is twice as large as the \( y \)-value on the graph of \( h(x) = 3x + 15 \). Approximate this value of \( x \) using Newton's Method with an initial estimate of \( x_1 = 1 \). You should use this method 3 times in order to obtain estimates \( x_2, x_3 \) and \( x_4 \). You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

\[
g'(x) = 2h'(x)
\]

\[
2x^3 + 10x + 9 = 2(3x + 15)
\]

\[
2x^3 + 4x - 21 = 0
\]

Let \( f(x) = 2x^3 + 4x - 21 \) and apply Newton's Method.

\[
f(x) = 2x^3 + 4x - 21 \Rightarrow f'(x) = 6x^2 + 4
\]

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n^3 + 4x_n - 21}{6x_n^2 + 4}
\]

\[x_1 = 1\] given as first estimate

\[
x_2 = x_1 - \frac{2x_1^3 + 4x_1 - 21}{6x_1^2 + 4} = 1 - \frac{2(1)^3 + 4(1) - 21}{6(1)^2 + 4} = 2.5 \quad x_2 = 2.5
\]

\[
x_3 \approx 2.012048193
\]

\[
x_4 \approx 2.012048193 - \frac{2(2.012048193)^3 + 4(2.012048193) - 21}{6(2.012048193)^2 + 4} \approx 2.012048193
\]

\[
x_4 \approx 2.012048193
\]

\[
x_5 \approx 1.894017385
\]

\[
x_6 \approx 1.887556998
\]

\[
x_7 \approx 1.887538329
\]

\[
x_7 \approx 1.887538329
\]

\[
x_7 \approx 1.887538329
\]