Math 220 **Quiz 12 (take-home)** **Fall 2018**

Name Solutions

(circle your TA discussion section)

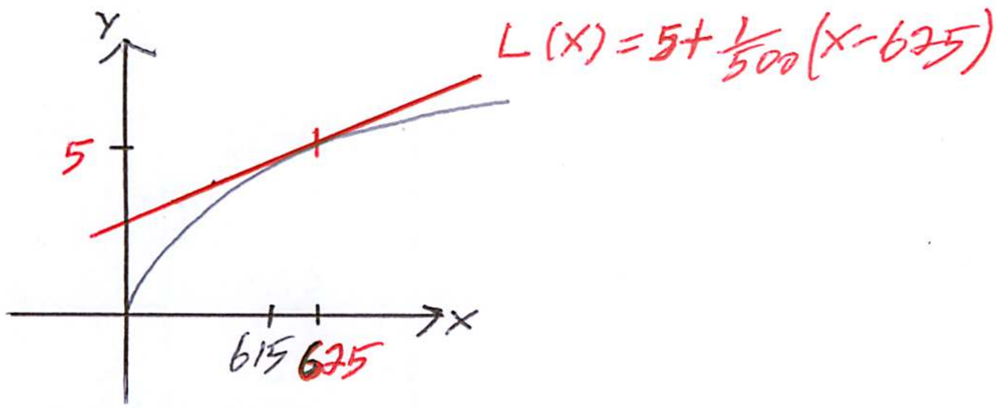
- ▹ **AD1**, TR 11:00-12:50, Adriana Morales
- ▹ **AD2**, TR 9:00-10:50, Hannah Burson
- ▹ **AD3**, TR 1:00-2:50, Dana Neidinger
- ▹ **ADA**, TR 8:00-8:50, Gayana Jayasinghe
- ▹ **ADB**, TR 9:00-9:50, Felix Clemen
- ▹ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▹ **ADD**, TR 11:00-11:50, Gidon Orelowitz
- ▹ **ADE**, TR 12:00-12:50, Josh Wen
- ▹ **ADF**, TR 1:00-1:50, Nachiketa Adhikari
- ▹ **ADG**, TR 2:00-2:50, Lutian Zhao
- ▹ **ADH**, TR 3:00-3:50, Stathis Chrontsios
- ▹ **ADI**, TR 4:00-4:50, Stathis Chrontsios
- ▹ **ADJ**, TR 9:00-9:50, Gayana Jayasinghe
- ▹ **ADK**, TR 10:00-10:50, Madina Bolat
- ▹ **ADL**, TR 11:00-11:50, Chris Loa
- ▹ **ADM**, TR 12:00-12:50, Heeyeon Kim
- ▹ **ADN**, TR 1:00-1:50, Josh Wen
- ▹ **ADO**, TR 2:00-2:50, Kesav Krishnan
- ▹ **ADQ**, TR 10:00-10:50, Felix Clemen
- ▹ **ADR**, TR 9:00-9:50, Madina Bolat
- ▹ **ADS**, TR 12:00-12:50, Chris Loa
- ▹ **ADT**, TR 2:00-2:50, Nachiketa Adhikari
- ▹ **ADU**, TR 3:00-3:50, Kesav Krishnan
- ▹ **ADZ**, TR 9:00-9:50, Gidon Orelowitz

- You may lose points if you do not circle your correct discussion section.
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page including old test and quiz solutions.
- The only computational technology allowed is a calculator for basic arithmetic ($+$, $-$, $\times$, $\div$).
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your lecture period on Friday, November 30th.**
- **TAs and Tutors – Do not help students with these specific problems until the quizzes have been collected for all MATH 220 lectures (10am, 1pm, 3pm).**

1. (3 points) A calculator gives an estimate of 4.979878868 for the value of $\sqrt[4]{615}$.

Using the techniques of linear approximation found in section 3.10, show that it is possible to obtain a very similar estimate of 4.98 without the use of any technology.

$f(x) = \sqrt[4]{x} = x^{1/4} \Rightarrow f'(x) = \frac{1}{4}x^{-3/4}$



tangent line at $x = 625$

point: $(625, \sqrt[4]{625}) = (625, 5)$

slope: $f'(625) = \frac{1}{4}(625)^{-3/4}$

$= \frac{1}{4(625^{1/4})^3}$

$= \frac{1}{4(5)^3}$

$= \frac{1}{500}$

$y - 5 = \frac{1}{500}(x - 625)$

$y = 5 + \frac{1}{500}(x - 625)$

$f(x) \approx L(x)$ near point of tangency

$\sqrt[4]{x} \approx 5 + \frac{1}{500}(x - 625)$ near $x = 625$

$\sqrt[4]{615} \approx 5 + \frac{1}{500}(615 - 625)$

$\sqrt[4]{615} \approx 5 - \frac{10}{500}$

$\sqrt[4]{615} \approx 4.98$

(seen as overestimate from graphs)

2. (3 points) Suppose that $f(x)$ and $g(x)$ are differentiable everywhere and satisfy the following conditions.

- $f(x)$ is an odd function
- $g(x)$ is an even function
- $f(-2) = 8$
- $g(-2) = -3$
- $f'(2) = 1/6$
- $g'(2) = 1/4$

Let $w(x) = f(x)g(x)$. Use the properties of even and odd functions along with the techniques of linear approximation found in section 3.10 to estimate the value of $w(2.2)$. Simplify your answer.

f is odd and $f(-2)=8 \Rightarrow f(2)=-8$

g is even and $g(-2)=-3 \Rightarrow g(2)=-3$

$w(x)=f(x)g(x) \Rightarrow w'(x)=f'(x)g(x)+f(x)g'(x)$

$w(2)=f(2)g(2)=(-8)(-3)=24$

$w'(2)=f'(2)g(2)+f(2)g'(2)=\left(\frac{1}{6}\right)(-3)+(-8)\left(\frac{1}{4}\right)=-\frac{5}{2}$

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point: $(2,24)$

slope: $-\frac{5}{2}$

tang. line: $y-24=-\frac{5}{2}(x-2)$

$y=24-\frac{5}{2}(x-2)$

(linear approximation using tangent line to $w(x)$ at $x=2$)

$w(x)\approx 24-\frac{5}{2}(x-2)$ for x near 2

$w(2.2)\approx 24-\frac{5}{2}(2.2-2)$

$w(2.2)\approx 23.5$

② Another estimate using linear approximation

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f is odd and $f(-2)=8 \Rightarrow f(2)=-8$ (point: $(2,-8)$)

$f'(2)=1/6$ (slope: $\frac{1}{6}$)

$y-(-8)=\frac{1}{6}(x-2)$

$y=-8+\frac{1}{6}(x-2)$

(lin. approx. for $f(x)$ near 2)

$f(x) \approx -8+\frac{1}{6}(x-2)$ for x near 2

$f(2.2) \approx -8+\frac{1}{6}(2.2-2)$

$f(2.2) \approx \frac{-239}{30} = -7.9666...$

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g is even and $g(-2)=-3 \Rightarrow g(2)=-3$ (point: $(2,-3)$)

$g'(2)=1/4$ (slope: $\frac{1}{4}$)

$y-(-3)=\frac{1}{4}(x-2)$

$y=-3+\frac{1}{4}(x-2)$

(lin. approx. for $g(x)$ near 2)

$g(x) \approx -3+\frac{1}{4}(x-2)$ for x near 2

$g(2.2) \approx -3+\frac{1}{4}(2.2-2)$

$g(2.2) \approx -\frac{59}{20} = -2.95$

$w(x)=f(x)g(x)$

$w(2.2)=f(2.2)g(2.2)=\left(\frac{-239}{30}\right)\left(\frac{-59}{20}\right) \approx 23.50167$

3. (4 points) There is one value of x for which the y-value on the graph of $g(x) = 2x^3 + 10x + 9$ is twice as large as the y-value on the graph of $h(x) = 3x + 15$. Approximate this value of x using Newton's Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$$\left.\begin{aligned} g(x) &= 2\cdot h(x) \\ 2x^3+10x+9 &= 2(3x+15) \\ 2x^3+4x-21 &= 0 \end{aligned}\right\} \text{let } f(x) = 2x^3+4x-21 \text{ and apply Newton's Method}$$

$$f(x) = 2x^3+4x-21 \Rightarrow f'(x) = 6x^2+4$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{2x_n^3+4x_n-21}{6x_n^2+4}$$

$x_1 = 1$ given as first estimate

$$x_2 = x_1 - \frac{2x_1^3+4x_1-21}{6x_1^2+4} = 1 - \frac{2(1)^3+4(1)-21}{6(1)^2+4} = 2.5 \quad \boxed{x_2 = 2.5}$$

$$x_3 = x_2 - \frac{2x_2^3+4x_2-21}{6x_2^2+4} = 2.5 - \frac{2(2.5)^3+4(2.5)-21}{6(2.5)^2+4} \approx 2.012048193$$

$x_3 \approx 2.012048193$

$$x_4 = x_3 - \frac{2x_3^3+4x_3-21}{6x_3^2+4}$$

$$x_4 \approx 2.012048193 - \frac{2(2.012048193)^3+4(2.012048193)-21}{6(2.012048193)^2+4}$$

$x_4 \approx 1.894017385$

$$\left.\begin{aligned} x_5 &\approx 1.887556998 \\ x_6 &\approx 1.887538329 \\ x_7 &\approx 1.887538329 \end{aligned}\right) \text{ a few extra estimates}$$