Math 220 (section AD?) **Quiz 11** **Fall 2018**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{12}{4x^2+1}$ on the interval $[0, 1/2]$. Simplify your answer.

$$f_{ave} = \frac{1}{\frac{1}{2}-0}\int_0^{1/2} \frac{12}{4x^2+1}\,dx$$

$$= 2\int_0^{1/2} \frac{12}{(2x)^2+1}\,dx$$

$$= 2\int_0^{1} \frac{6}{u^2+1}\,du$$

$$= 12\int_0^{1} \frac{1}{u^2+1}\,du$$

$$= 12\Big[\arctan(u)\Big]_0^1$$

$$= 12\left[\arctan(1) - \arctan(0)\right]$$

$$= 12\left[\frac{\pi}{4} - 0\right]$$

$$= 3\pi$$

substitution

$u = 2x$

$du = 2dx$

$6du = 12dx$

$x = 0 \Rightarrow u = 2\cdot 0 = 0$

$x = \frac{1}{2} \Rightarrow u = 2\cdot\frac{1}{2} = 1$

2. Let **R** be the finite region bounded by the graphs determined by the following equations.

$y = 5\ln(x)$

$x = e^2$

$y = 15$

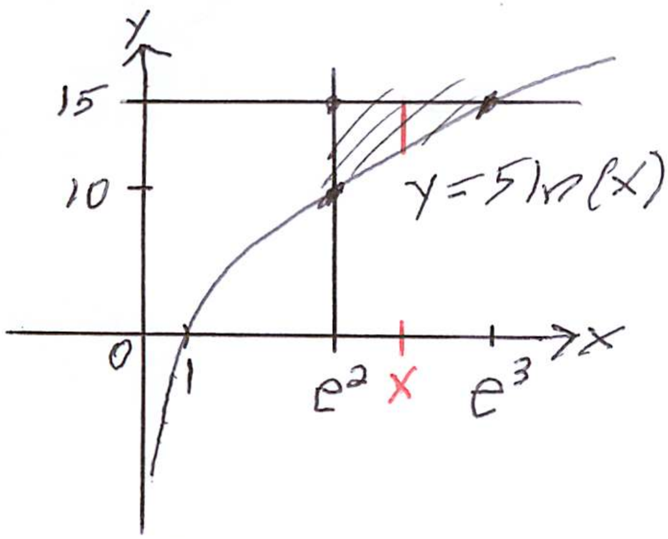
intersections

$5\ln(x) = 15 \Rightarrow \ln(x) = 3 \Rightarrow x = e^3$

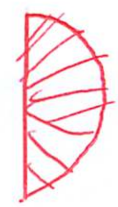
at $x = e^2$, $y = 5\ln(e^2) = 5 \cdot 2 = 10$

Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are semicircles.



cross-section at x is a semicircle



$$V = \int_{e^2}^{e^3} (\text{cross-sectional area})\, dx$$

$$V = \int_{e^2}^{e^3} \frac{1}{2}\pi r^2\, dx$$

$$V = \int_{e^2}^{e^3} \frac{1}{2}\pi \left(\frac{\text{diameter}}{2}\right)^2 dx$$

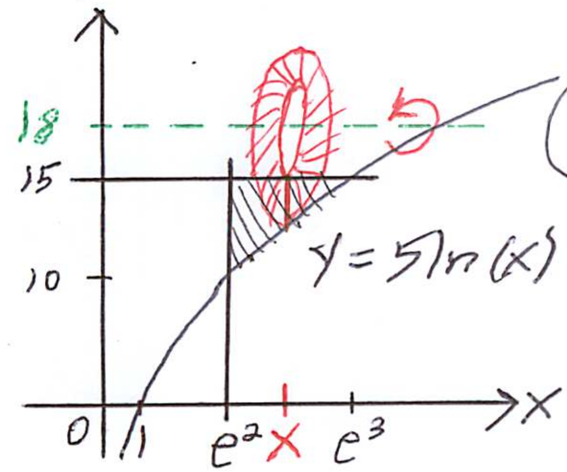
$$V = \int_{e^2}^{e^3} \frac{1}{2}\pi \left(\frac{15 - 5\ln(x)}{2}\right)^2 dx$$

(b) The volume of the solid formed when **R** is revolved around the line $y = 18$. Set up the integrals for this volume in the following two ways.

i. (2 points) Integrate with respect to x.

$$V = \int_{e^2}^{e^3} \left(\begin{matrix}\text{cross-sectional} \\ \text{area}\end{matrix}\right) dx$$

$$V = \int_{e^2}^{e^3} \left(\pi r_{out}^2 - \pi r_{in}^2\right) dx$$



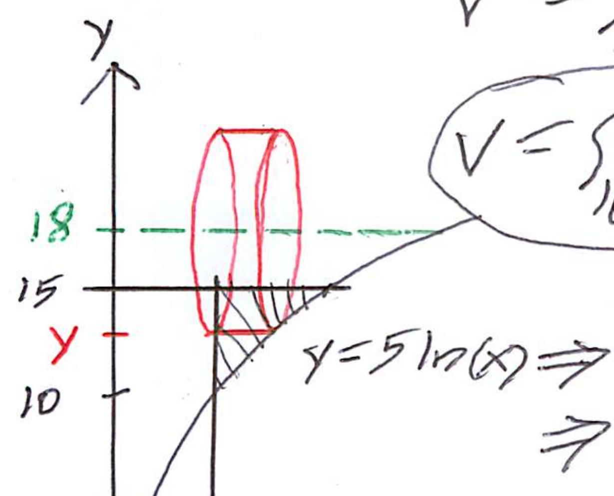
$$\boxed{V = \int_{e^2}^{e^3} \left(\pi(18 - 5\ln(x))^2 - \pi(18-15)^2\right) dx}$$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)

$$V = \int_{10}^{15} \left(\begin{matrix}\text{surface} \\ \text{area}\end{matrix}\right) dy$$

$$V = \int_{10}^{15} 2\pi r h \, dy$$

$$\boxed{V = \int_{10}^{15} 2\pi(18 - y)(e^{y/5} - e^2) dy}$$



$$y = 5\ln(x) \Rightarrow \ln(x) = \frac{y}{5}$$
$$\Rightarrow x = e^{y/5}$$