1. (3 points) Find the average value of the function \( f(x) = \frac{12}{4x^2 + 1} \) on the interval \([0, 1/2]\). Simplify your answer.

\[
\frac{f_{\text{ave}}}{2} = \frac{1}{\frac{1}{2} - 0} \int_{0}^{\frac{1}{2}} \frac{12}{4x^2 + 1} \, dx \\
= 2 \int_{0}^{\frac{1}{2}} \frac{12}{(2x)^2 + 1} \, dx \\
= 2 \int_{0}^{1} \frac{6}{u^2 + 1} \, du \\
= 12 \int_{0}^{1} \frac{1}{u^2 + 1} \, du \\
= 12 \left[ \arctan(u) \right]_{0}^{1} \\
= 12 \left[ \arctan(1) - \arctan(0) \right] \\
= 12 \left[ \frac{\pi}{4} - 0 \right] \\
= \sqrt{3} \pi
\]
2. Let \( R \) be the finite region bounded by the graphs determined by the following equations.

\[
\begin{align*}
y &= 5 \ln(x) \\
x &= e^2 \\
y &= 15
\end{align*}
\]

\[
\text{intersections} \\
5 \ln(x) = 15 \implies \ln(x) = 3 \implies x = e^3 \\
\text{at } x = e^2, \ y = 5 \ln(e^2) = 5 \cdot 2 = 10
\]

Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base \( R \) for which the cross-sections perpendicular to the \( x \)-axis are semicircles.

\[
V = \int_{e^2}^{e^3} \left( \text{cross-sections area} \right) dx
\]

\[
V = \int_{e^2}^{e^3} \frac{1}{2} \pi r^2 \; dx
\]

\[
V = \int_{e^2}^{e^3} \frac{1}{2} \pi \left( \frac{\text{diameter}}{2} \right)^2 \; dx
\]

\[
V = \int_{e^2}^{e^3} \frac{1}{2} \pi \left( \frac{15 - 5 \ln(x)}{2} \right)^2 \; dx
\]
(b) The volume of the solid formed when \( R \) is revolved around the line \( y = 18 \). Set up the integrals for this volume in the following two ways.

i. (2 points) Integrate with respect to \( x \).

\[
V = \int \frac{e^3}{e^2} \left( \text{cross-sectional area} \right) \, dx
\]

\[
V = \int \frac{e^3}{e^2} \left( \pi r_{\text{out}}^2 - \pi r_{\text{in}}^2 \right) \, dx
\]

\[
V = \int \frac{e^3}{e^2} \left( \pi \left( 18 - \ln(x) \right)^2 - \pi \left( 18 - 15 \right)^2 \right) \, dx
\]

ii. (2 points) Integrate with respect to \( y \). (Use different integrands in parts i and ii.)

\[
V = \int_{10}^{15} \left( \text{surface area} \right) \, dy
\]

\[
V = \int_{10}^{15} 2\pi r h \, dy
\]

\[
V = \int_{10}^{15} 2\pi (18-y)(e^{y/15} - e^2) \, dy
\]

\[y = 5 \ln(x) \Rightarrow \ln(x) = \frac{18}{5} \\Rightarrow x = e^{18/5}\]