Math 220 (section AD?) **Quiz 10** **Fall 2018**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Fill in the missing information for the following two theorems.

Mean Value Theorem Let f be a function that satisfies the following two hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Rolle's Theorem Let f be a function that satisfies the following three hypotheses.

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

(3) $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_{\sqrt{17}}^{\sqrt{41}} \frac{10x}{\sqrt{x^2+8}}\,dx = \int_{\sqrt{17}}^{\sqrt{41}} \frac{5}{\sqrt{x^2+8}} \cdot 2x\,dx$$

Substitution

$u = x^2+8$

$du = 2x\,dx$

$x = \sqrt{17} \Rightarrow u = (\sqrt{17})^2+8 = 25$

$x = \sqrt{41} \Rightarrow u = (\sqrt{41})^2+8 = 49$

$$= \int_{25}^{49} \frac{5}{\sqrt{u}}\,du$$

$$= \int_{25}^{49} 5u^{-1/2}\,du$$

$$= \left[10u^{1/2}\right]_{25}^{49}$$

$$= 10\sqrt{49} - 10\sqrt{25}$$

$$= 70 - 50$$

$$= 20$$

3. (2 points) Evaluate the indefinite integral.

$$\int \frac{10x^4}{x^{10}+1}\,dx = \int \frac{2}{(x^5)^2+1} \cdot 5x^4\,dx$$

Substitution

$u = x^5$

$du = 5x^4\,dx$

$$= \int \frac{2}{u^2+1}\,du$$

$$= 2\int \frac{1}{u^2+1}\,du$$

$$= 2\arctan(u) + C$$

$$= 2\arctan(x^5) + C$$

4. (2 points) Evaluate the indefinite integral.

$$\int e^{6x}\left(e^{3x}+1\right)^{20}\,dx = \int e^{3x}(e^{3x}+1)^{20}e^{3x}\,dx$$

substitution

$u = e^{3x}+1 \Rightarrow u-1 = e^{3x}$

$du = 3e^{3x}dx \Rightarrow \frac{1}{3}du = e^{3x}dx$

$$= \int (u-1)\,u^{20}\,\frac{1}{3}\,du$$

$$= \frac{1}{3}\int (u^{21}-u^{20})\,du$$

$$= \frac{1}{3}\left(\frac{1}{22}u^{22}-\frac{1}{21}u^{21}\right)+C$$

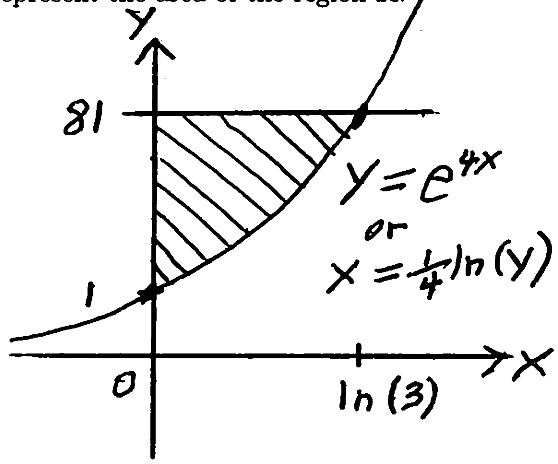
$$= \frac{1}{66}(e^{3x}+1)^{22}-\frac{1}{63}(e^{3x}+1)^{21}+C$$

5. (2 points) Let **R** be the finite region bounded by the given functions. In the following way, set up <u>but do not evaluate</u> definite integrals which represent the area of the region **R**.

$y = e^{4x}$

$y = 81$

$x = 0$



$y = e^{4x} \Rightarrow \ln(y) = \ln(e^{4x})$

$\Rightarrow \ln(y) = 4x$

$\Rightarrow x = \frac{1}{4}\ln(y)$

intersection: $e^{4x} = 81 \Rightarrow \ln(e^{4x}) = \ln(81)$

$\Rightarrow 4x = \ln(81)$

$\Rightarrow x = \frac{1}{4}\ln(81) = \frac{1}{4}\ln(3^4) = \ln(3)$

(a) Integrate with respect to x.

$$\text{area} = \int_{x_{min}}^{x_{max}} (y_{top} - y_{bottom})\,dx$$

$$\text{area} = \int_{0}^{\ln(3)} (81 - e^{4x})\,dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$\text{area} = \int_{y_{min}}^{y_{max}} (x_{right} - x_{left})\,dy$$

$$\text{area} = \int_{1}^{81} \left(\tfrac{1}{4}\ln(y) - 0\right)\,dy$$