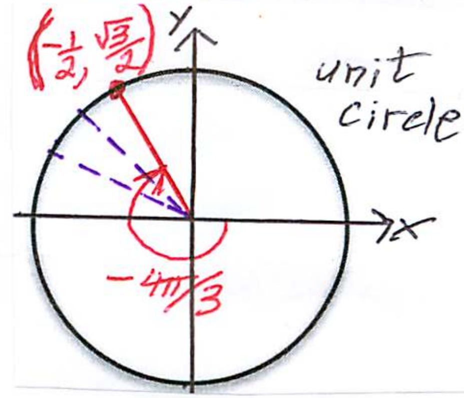
Math 220 (section AD?) Quiz 1 Fall 2018

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

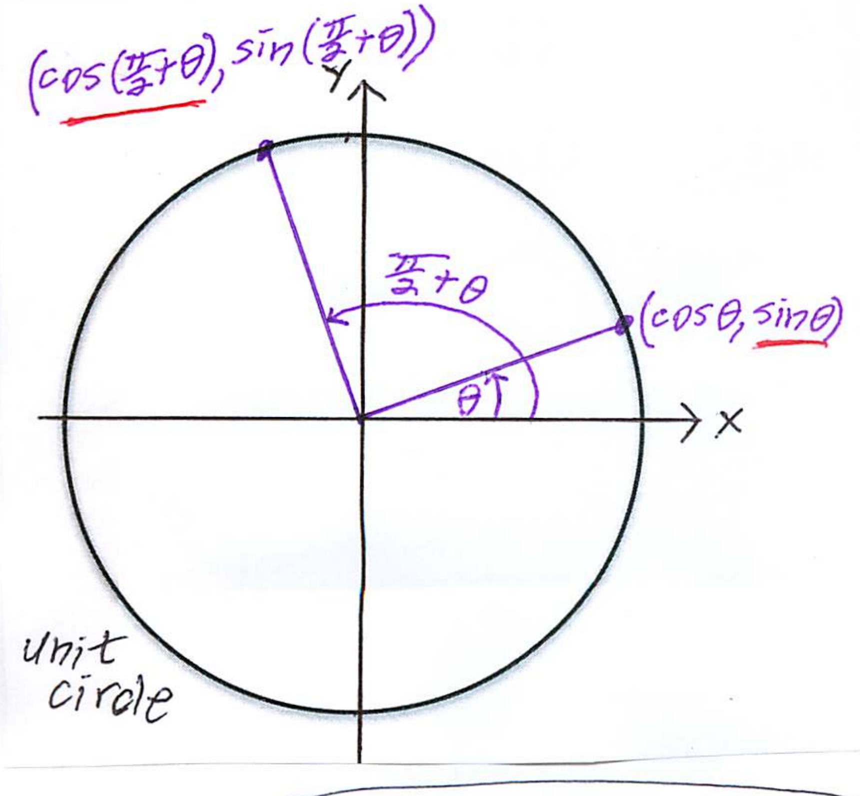


1. (1 point) Evaluate $\cot\left(\frac{-4\pi}{3}\right)$.

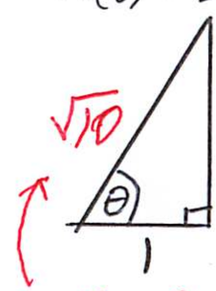
$$\cot(-4\pi/3) = \frac{\cos(-4\pi/3)}{\sin(-4\pi/3)}$$

$$= \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

2. (3 points) Given an acute angle θ for which $\tan(\theta) = 3$, evaluate $\cos\left(\frac{\pi}{2} + \theta\right)$.



$\tan(\theta) = 3 = \frac{3}{1}$ $\left(\frac{opp}{adj}\right)$



$1^2 + 3^2 = (hyp)^2$ (pythagorean theorem)

$hyp = \sqrt{10}$

$\sin(\theta) = \frac{3}{\sqrt{10}}$ $\left(\frac{opp}{hyp}\right)$

$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

$$= -\frac{3}{\sqrt{10}}$$

$$= \frac{-3\sqrt{10}}{10}$$

3. (3 points) Determine the domain of the given function.

$$f(x) = \frac{\sqrt{3\sin(x) + 10}}{\sqrt{7-x} - \sqrt{2x-5}}$$

① $\sin(x) \in [-1,1] \Rightarrow 3\sin(x)+10 \in [7,13]$. No restriction on domain,

② From $\sqrt{7-x}$, $7-x \geq 0 \Rightarrow x \leq 7$

③ From $\sqrt{2x-5}$, $2x-5 \geq 0 \Rightarrow x \geq 5/2$

④ Denominator $= 0$ when $\sqrt{7-x} - \sqrt{2x-5} = 0$

$$\sqrt{7-x} = \sqrt{2x-5}$$
$$7-x = 2x-5$$
$$-3x = -12$$
$$x = 4$$

Thus $x \neq 4$

Domain of $f(x)$: $[\frac{5}{2}, 4) \cup (4, 7]$

4. (3 points) Use the definitions of even and odd functions to prove whether the following function is even, odd or neither.

$$f(x) = x^5\cos\left(x^3\right) + x^4\sin(x)$$

$$f(-x) = (-x)^5\cos((-x)^3) + (-x)^4\sin(-x)$$
$$= -x^5\cos(-x^3) + x^4(-\sin(x)) \quad \text{(since sin is odd)}$$
$$= -x^5\cos(x^3) - x^4\sin(x) \quad \text{(since cos is even)}$$
$$= -\left(x^5\cos(x^3) + x^4\sin(x)\right)$$
$$= -f(x)$$

Therefore $f(x)$ is an odd function.