Math 220 (section AD?)  

Quiz 1  

Fall 2018  

Name: SOLUTIONS

- You have 20 minutes  
- No calculators  
- Show sufficient work

1. (1 point) Evaluate \( \cot \left( -\frac{4\pi}{3} \right) \).

\[
\cot \left( -\frac{4\pi}{3} \right) = \frac{\cos \left( -\frac{4\pi}{3} \right)}{\sin \left( -\frac{4\pi}{3} \right)} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}
\]

2. (3 points) Given an acute angle \( \theta \) for which \( \tan(\theta) = 3 \), evaluate \( \cos \left( \frac{\pi}{2} + \theta \right) \).

\[
\tan(\theta) = 3 = \frac{3}{1} \quad \text{(opp/adj)}
\]

\[
1^2 + 3^2 = (\text{hyp})^2 \quad \text{(Pythagorean theorem)}
\]

\[
\text{hyp} = \sqrt{10}
\]

\[
\sin(\theta) = \frac{3}{\sqrt{10}} \quad \text{(opp/hyp)}
\]

\[
\cos \left( \frac{\pi}{2} + \theta \right) = -\sin(\theta) = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}
\]
3. (3 points) Determine the domain of the given function.

\[ f(x) = \frac{\sqrt{3} \sin(x) + 10}{\sqrt{7-x} - \sqrt{2x-5}} \]

\[ \sin(x) \in [-1, 1] \Rightarrow 3\sin(x) + 10 \in [7, 13] \]. No restriction on domain.

2. From \( \sqrt{7-x} \), \( 7-x \geq 0 \) \( \Rightarrow x \leq 7 \)

3. From \( \sqrt{2x-5} \), \( 2x-5 \geq 0 \) \( \Rightarrow x \geq \frac{5}{2} \)

4. Denominator = 0 when \( \sqrt{7-x} - \sqrt{2x-5} = 0 \)
   \[ \sqrt{7-x} = \sqrt{2x-5} \]
   \[ 7-x = 2x-5 \]
   \[ -3x = -12 \]
   \[ x = 4 \]

Thus \( x \neq 4 \)

Domain of \( f(x) \): \( \left[ \frac{5}{2}, 4 \right) \cup (4, 7] \)

4. (3 points) Use the definitions of even and odd functions to prove whether the following function is even, odd or neither.

\[ f(x) = x^5 \cos(x^3) + x^4 \sin(x) \]

\[ f(-x) = (-x)^5 \cos((-x)^3) + (-x)^4 \sin(-x) \]

\[ = -x^5 \cos(-x^3) + x^4 (-\sin(x)) \]

\[ = -x^5 \cos(x^3) - x^4 \sin(x) \]

\[ = -(x^5 \cos(x^3) + x^4 \sin(x)) \]

Therefore \( f(x) \) is an **odd** function.