Math 220 (section AD?) **Quiz 9** **Fall 2016**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) At time t hours, a population of bacteria is growing at a rate of $20t + 50$ bacteria per hour. If the population is 2000 at time $t = 1$, then what is the population at time $t = 3$ hours?

$$\begin{pmatrix}\text{Pop. at}\\ t=3\end{pmatrix} = \begin{pmatrix}\text{Pop. at}\\ t=1\end{pmatrix} + \begin{pmatrix}\text{net change in pop.}\\ \text{between } t=1 \text{ and } t=3\end{pmatrix}$$

$$= 2000 + \int_1^3 (20t+50)\,dt$$

$$= 2000 + \left[10t^2+50t\right]_1^3$$

$$= 2000 + (10\cdot 3^2+50\cdot 3) - (10\cdot 1^2+50\cdot 1)$$

$$= 2000 + 240 - 60 = 2180 \text{ Bacteria}$$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_1^9 \frac{15\sqrt{x} - 6x^2}{3x}\,dx = \int_1^9 \left(\frac{15\sqrt{x}}{3x} - \frac{6x^2}{3x}\right) dx$$

$$= \int_1^9 (5x^{-1/2} - 2x)\,dx$$

$$= \left[10x^{1/2} - x^2\right]_1^9$$

$$= (10\sqrt{9} - 9^2) - (10\sqrt{1} - 1^2)$$

$$= -51 - 9$$

$$= -60$$

3. (2 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{x^4+2x^2+6}{x^2+1}\,dx$ $= \int \left(x^2+1+\frac{5}{x^2+1}\right)dx$

$= \frac{1}{3}x^3 + x + 5\arctan(x) + C$

POLYNOMIAL LONG DIVISION

$$\begin{array}{r} x^2+1 \\ x^2+1 \overline{\smash{)}\, x^4+2x^2+6} \\ \underline{x^4+x^2} \\ x^2+6 \\ \underline{x^2+1} \\ 5 \end{array}$$

$\Rightarrow \frac{x^4+2x^2+6}{x^2+1} = x^2+1+\frac{5}{x^2+1}$

(b) $\int \frac{\sin(2x)}{\sin(x)\cot(x)}\,dx$ $= \int \frac{2\sin(x)\cos(x)}{\sin(x)\cdot\frac{\cos(x)}{\sin(x)}}\,dx$

$= \int 2\sin(x)\,dx$

$= -2\cos(x) + C$

4. (2 points) Suppose $p(x) = \int_5^{x^3} \tan\left(t^2\right)\, dt$. Find its second derivative $p''(x)$.

From part 1 of the Fundamental Theorem of Calculus along with the Chain Rule,

$$p'(x) = \tan((x^3)^2) \cdot \frac{d}{dx}(x^3)$$

$$p'(x) = \tan(x^6) \cdot 3x^2$$

$$p''(x) = \frac{d}{dx}(\tan(x^6)) \cdot 3x^2 + \tan(x^6) \cdot \frac{d}{dx}(3x^2)$$

$$p''(x) = \sec^2(x^6) \cdot 6x^5 \cdot 3x^2 + \tan(x^6) \cdot 6x$$

$$p''(x) = 18x^7 \sec^2(x^6) + 6x \tan(x^6)$$