Math 220 **Quiz 7 (take-home)** **Fall 2016**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Andrew McConvey
- ▷ **AD2**, TR 9:00-10:50, Ben Wright
- ▷ **AD3**, TR 1:00-2:50, Cassie Christenson
- ▷ **ADA**, TR 8:00-8:50, Alexi Block Gorman
- ▷ **ADB**, TR 9:00-9:50, Dakota Ihli
- ▷ **ADC**, TR 10:00-10:50, Elizabeth Field
- ▷ **ADD**, TR 11:00-11:50, Adam Wagner
- ▷ **ADE**, TR 12:00-12:50, Adam Wagner
- ▷ **ADF**, TR 1:00-1:50, Tsutomu Okano
- ▷ **ADG**, TR 2:00-2:50, Xujun 'Henry' Liu
- ▷ **ADH**, TR 3:00-3:50, Mychael Sanchez
- ▷ **ADI**, TR 4:00-4:50, Mychael Sanchez
- ▷ **ADJ**, TR 9:00-9:50, Kyle Pratt
- ▷ **ADK**, TR 10:00-10:50, Kyle Pratt
- ▷ **ADL**, TR 11:00-11:50, Tigran Hakobyan
- ▷ **ADM**, TR 12:00-12:50, Liz Tatum
- ▷ **ADN**, TR 1:00-1:50, Xujun 'Henry' Liu
- ▷ **ADO**, TR 2:00-2:50, Tigran Hakobyan
- ▷ **ADP**, TR 3:00-3:50, Liz Tatum
- ▷ **ADQ**, TR 10:00-10:50, Dakota Ihli
- ▷ **ADR**, TR 9:00-9:50, Elizabeth Field
- ▷ **ADS**, TR 12:00-12:50, Tsutomu Okano
- ▷ **ADT**, TR 2:00-2:50, Anna Weigandt
- ▷ **ADU**, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official lecture period on Friday, October 14.**
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until the quizzes have been collected for all of my lectures (9am, noon, 1pm).**

1. (2 points) Evaluate $\lim_{x\to\infty}\left(1+\frac{2}{3x^2}\right)^{6x^2}$ (Indeterminate form 1^∞)

$$\lim_{x\to\infty}\left(1+\frac{2}{3x^2}\right)^{6x^2} = \lim_{x\to\infty} e^{\ln\left(\left(1+\frac{2}{3x^2}\right)^{6x^2}\right)}$$

$$= e^{\lim_{x\to\infty}\ln\left(\left(1+\frac{2}{3x^2}\right)^{6x^2}\right)}$$ (okay since e^x is cont. everywhere)

$$= e^{\lim_{x\to\infty}\left(6x^2\cdot\ln\left(1+\frac{2}{3x^2}\right)\right)}$$ (Indet. form $\infty\cdot 0$)

$$= e^{\lim_{x\to\infty}\frac{6\ln\left(1+\frac{2}{3x^2}\right)}{1/x^2}}$$ $\to 0$, $\to 0$ (Indet. form $\frac{0}{0}$)

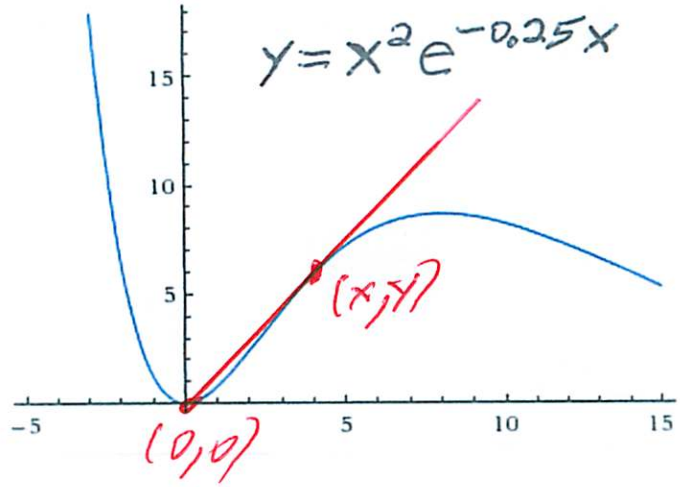
$$= e^{\lim_{x\to\infty}\left(\frac{\frac{d}{dx}\left(6\ln\left(1+\frac{2}{3x^2}\right)\right)}{\frac{d}{dx}\left(1/x^2\right)}\right)}$$ (by l'Hospital's Rule)

$$= e^{\lim_{x\to\infty}\left(\frac{6\cdot\frac{1}{1+\frac{2}{3x^2}}\cdot -\frac{4}{3}x^{-3}}{-2x^{-3}}\right)}$$

$$= e^{\lim_{x\to\infty}\left(\frac{4}{1+\frac{2}{3x^2}}\right)}$$

$$= e^4$$

2. (3 points) For each $x > 0$, let $m(x)$ be the slope of the line which goes through the point $(0,0)$ and the point (x,y) on the curve $y = x^2e^{-0.25x}$.



What is the largest possible value for $m(x)$?

$$\text{slope} = \frac{\Delta y}{\Delta x}$$
$$= \frac{y-0}{x-0}$$
$$= \frac{x^2e^{-0.25x}-0}{x-0}$$
$$= xe^{-0.25x}$$

we must maximize $m(x) = xe^{-0.25x}$ for $x>0$

$$m'(x) = \frac{d}{dx}(x)\cdot e^{-0.25x} + x\cdot\frac{d}{dx}(e^{-0.25x})$$
$$= 1\cdot e^{-0.25x} + x\cdot -0.25e^{-0.25x}$$
$$= \underbrace{e^{-0.25x}}_{\text{always positive}}\underbrace{(1-0.25x)}_{\text{equals } 0 \text{ when } x=4}$$

The only critical number is $x=4$

Values of $m'(x)$

$$\begin{array}{ccccc} & +++ & 0 & --- & \\ 0 & & 4 & & \to x \end{array}$$

The maximum value of $m(x)$ occurs for $x=4$

largest slope of such a line is $m(4) = 4\cdot e^{-0.25(4)}$
$$= 4e^{-1}$$
$$= \frac{4}{e}$$

3. (3 points) What are the coordinates (x, y) for the highest point on the graph of the function $f(x) = \dfrac{e^{6x}}{e^{9x}+4}$?

maximize $f(x) = \dfrac{e^{6x}}{e^{9x}+4}$ for $-\infty < x < \infty$

$$f'(x) = \frac{6e^{6x}(e^{9x}+4) - e^{6x}(9e^{9x})}{(e^{9x}+4)^2}$$

$$= \frac{3e^{6x}[2(e^{9x}+4) - 3e^{9x}]}{(e^{9x}+4)^2}$$

$$= \frac{3e^{6x}[8 - e^{9x}]}{(e^{9x}+4)^2}$$

Since $3e^{6x}$ and $(e^{9x}+4)^2$ are positive for all x,

The only critical number is when $8 - e^{9x} = 0$

That is $e^{9x} = 8 \Rightarrow 9x = \ln 8 \Rightarrow x = \frac{\ln 8}{9} = \frac{\ln(2^3)}{9}$

$= \frac{3\ln(2)}{9} = \frac{\ln(2)}{3}$

values of $f'(x)$

$+++ \quad 0 \quad - - - \; -$

$\xrightarrow{\hspace{6em}} x$

$\frac{\ln(2)}{3}$

highest point is $\left(\frac{\ln(2)}{3}, f\left(\frac{\ln(2)}{3}\right)\right)$

$$= \left(\frac{\ln(2)}{3}, \frac{e^{6\cdot\frac{\ln(2)}{3}}}{e^{9\cdot\frac{\ln(2)}{3}}+4}\right) = \left(\frac{\ln(2)}{3}, \frac{4}{12}\right)$$

$$= \left(\frac{\ln(2)}{3}, \frac{1}{3}\right)$$

4. (2 points) Complete the sentences concerning the function $f(x) = 3 + 4xe^{-5x}$.

(a) The function f is decreasing on the interval $[\frac{1}{5}, \infty)$

(b) The function f is increasing on the interval $(-\infty, \frac{1}{5}]$

(c) The function f is concave down on the interval $(-\infty, \frac{2}{5})$

(d) The function f is concave up on the interval $(\frac{2}{5}, \infty)$

$$f'(x) = 4e^{-5x} + 4xe^{-5x} \cdot (-5)$$
$$= 4e^{-5x}(1-5x)$$

$$f''(x) = 4e^{-5x} \cdot (-5)(1-5x) + 4e^{-5x} \cdot (-5)$$
$$= -20e^{-5x}(1-5x+1)$$
$$= -20e^{-5x}(2-5x)$$

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values of $f'(x)$

+++ 0 ---

f incr. $\frac{1}{5}$ f decr.

values of $f''(x)$

--- 0 +++

f conc. down $\frac{2}{5}$ f conc. up