Math 220 (section AD?) **Quiz 6** **Fall 2016**

Name: Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (4 points) The height of a remote-controlled drone in feet above ground for $t \geq 0$ seconds is given by the following function.

$$h(t) = 3t^2 e^{-t/10}$$

What is the maximum height obtained by the drone?

$$h'(t) = 6t \cdot e^{-t/10} + 3t^2 \cdot e^{-t/10} \cdot \left(-\tfrac{1}{10}\right)$$

$$h'(t) = 3te^{-t/10}\left(2 - \tfrac{1}{10}t\right)$$

$h'(t) = 0$ when $t = 0$ or $t = 20$

values of $h'(t)$

$- - - \; 0 \; + + + \; 0 \; - - -$

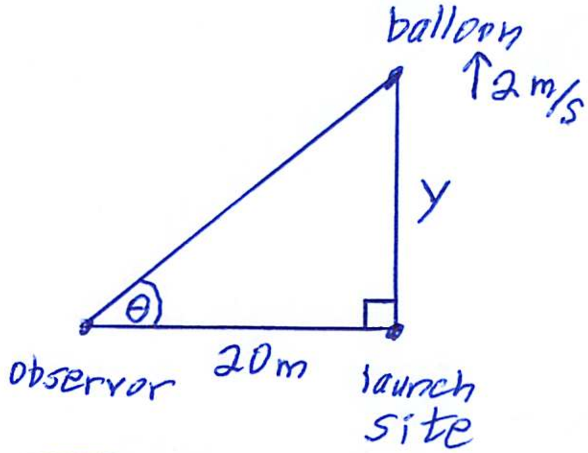
0 h incr. 20 h decr.

For $t \geq 0$, the maximum height is reached at $t = 20$ seconds

$$h(20) = 3 \cdot (20)^2 e^{-20/10}$$

$$= \frac{1200}{e^2} \text{ ft}$$

2. (3 points) There is a launch site of a hot-air balloon on the ground 20 meters away from an observor. The balloon rises vertically at a constant rate of 2 meters per second. How quickly is the angle of elevation of the balloon increasing 5 seconds after its launch?



Given) $\frac{dy}{dt} = 2\ m/s$

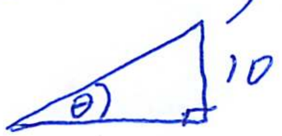
Want) $\left.\frac{d\theta}{dt}\right|_{t=5s}$

$\tan(\theta) = \frac{y}{20}$

$\frac{d}{dt}(\tan(\theta)) = \frac{d}{dt}\left(\frac{y}{20}\right)$

$\sec^2(\theta) \cdot \frac{d\theta}{dt} = \frac{1}{20} \cdot \frac{dy}{dt}$

At $t=5s$, we obtain $y = (2\ m/s)(5s) = 10\ m$



$\Rightarrow \sec(\theta) = \frac{hyp}{adj} = \frac{\sqrt{20^2+10^2}}{20} = \frac{\sqrt{500}}{20}$

$\left(\frac{\sqrt{500}}{20}\right)^2 \cdot \frac{d\theta}{dt} = \frac{1}{20} \cdot 2$

$\frac{500}{400} \cdot \frac{d\theta}{dt} = \frac{2}{20}$

$\frac{d\theta}{dt} = \frac{2}{20} \cdot \frac{400}{500} = \frac{2}{25}\ rad/s$

3. (3 points) Determine the formula for a function whose graph passes through the point $(6, e^{10})$ and has the property that for each point on the curve, the slope of the curve is equal to one half its y-coordinate.

$$\frac{dy}{dx} = \frac{1}{2}y \Rightarrow y = C \cdot e^{\frac{1}{2}x}$$

$$e^{10} = C \cdot e^{\frac{1}{2} \cdot 6}$$

$$e^{10} = Ce^{3}$$

$$C = \frac{e^{10}}{e^{3}}$$

$$C = e^{7}$$

$$y = e^{7}e^{\frac{1}{2}x}$$

$$y = e^{7+\frac{1}{2}x}$$