Math 220 (section AD?) Quiz 5 Fall 2016

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Compute the first derivative $v'(t)$ for the given function.

$$v(t) = \sin^4\left(\ln\left(t^8 + 1\right)\right)$$

$$v(t) = \left(\sin\left(\ln\left(t^8+1\right)\right)\right)^4$$

$$v'(t) = 4\left(\sin\left(\ln\left(t^8+1\right)\right)\right)^3 \cdot \frac{d}{dt}\left(\sin\left(\ln\left(t^8+1\right)\right)\right)$$

$$v'(t) = 4\sin^3\left(\ln\left(t^8+1\right)\right) \cdot \cos\left(\ln\left(t^8+1\right)\right) \cdot \frac{d}{dt}\left(\ln\left(t^8+1\right)\right)$$

$$v'(t) = 4\sin^3\left(\ln\left(t^8+1\right)\right) \cdot \cos\left(\ln\left(t^8+1\right)\right) \cdot \frac{1}{t^8+1} \cdot \frac{d}{dt}\left(t^8+1\right)$$

$$v'(t) = 4\sin^3\left(\ln\left(t^8+1\right)\right) \cdot \cos\left(\ln\left(t^8+1\right)\right) \cdot \frac{1}{t^8+1} \cdot 8t^7$$

2. (2 points) Compute the second derivative $h''(x)$ for the given function.

$$h(x) = e^{\tan(x)}$$

$h'(x) = e^{\tan(x)} \cdot \frac{d}{dx}(\tan(x))$

$h'(x) = e^{\tan(x)} \cdot \sec^2(x)$

$h''(x) = \frac{d}{dx}(e^{\tan(x)}) \cdot \sec^2(x) + e^{\tan(x)} \cdot \frac{d}{dx}(\sec^2(x))$

$h''(x) = (e^{\tan(x)} \cdot \sec^2(x)) \cdot \sec^2(x) + e^{\tan(x)} \cdot 2\sec(x) \cdot \frac{d}{dx}(\sec(x))$

$h''(x) = e^{\tan(x)} \sec^4(x) + e^{\tan(x)} \cdot 2\sec(x) \cdot \sec(x)\tan(x)$

$h''(x) = e^{\tan(x)} \sec^4(x) + 2e^{\tan(x)} \sec^2(x)\tan(x)$

3. (3 points) Find the equation of the line tangent to the given curve at the point $(1, 2)$.

$$x^3y + 2xy^3 = 18$$

$$\frac{d}{dx}\left(x^3 \cdot y + 2x \cdot y^3\right) = \frac{d}{dx}(18)$$

$$\frac{d}{dx}(x^3)\cdot y + x^3 \cdot \frac{d}{dx}(y) + \frac{d}{dx}(2x)\cdot y^3 + 2x \cdot \frac{d}{dx}(y^3) = 0$$

$$3x^2 \cdot y + x^3 \cdot \frac{dy}{dx} + 2 \cdot y^3 + 2x \cdot 3y^2 \cdot \frac{dy}{dx} = 0$$

plug in $(x,y) = (1,2)$ to get

$$6 + \frac{dy}{dx} + 16 + 24\frac{dy}{dx} = 0$$

$$25\frac{dy}{dx} = -22$$

$$\frac{dy}{dx} = \frac{-22}{25}$$

another approach is to solve and get

$$\frac{dy}{dx} = \frac{-3x^2y - 2y^3}{x^3 + 6xy^2}$$

now plug in $(x,y) = (1,2)$ to get $\frac{dy}{dx} = \frac{-22}{25}$

Point: $(1,2)$

slope: $-\frac{22}{25}$

tangent line: $y - 2 = \frac{-22}{25}(x-1)$

or

$$y = \frac{-22}{25}x + \frac{72}{25}$$

4. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

$$y = (2x+1)^{\arctan(x)}$$

Method 1) $\ln(y) = \ln\left((2x+1)^{\arctan(x)}\right)$

$$\ln(y) = \arctan(x) \cdot \ln(2x+1)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(\arctan(x) \cdot \ln(2x+1))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x^2+1} \cdot \ln(2x+1) + \arctan(x) \cdot \frac{1}{2x+1} \cdot 2$$

$$\frac{dy}{dx} = y\left(\frac{\ln(2x+1)}{x^2+1} + \frac{2\arctan(x)}{2x+1}\right)$$

$$\frac{dy}{dx} = (2x+1)^{\arctan(x)}\left(\frac{\ln(2x+1)}{x^2+1} + \frac{2\arctan(x)}{2x+1}\right)$$

Method 2) using $u = e^{\ln(u)}$ we get

$$y = (2x+1)^{\arctan(x)} = e^{\ln\left((2x+1)^{\arctan(x)}\right)}$$

$$y = e^{\arctan(x) \cdot \ln(2x+1)}$$

$$\frac{dy}{dx} = e^{\arctan(x) \cdot \ln(2x+1)} \cdot \frac{d}{dx}(\arctan(x) \cdot \ln(2x+1))$$

$$\frac{dy}{dx} = e^{\arctan(x) \cdot \ln(2x+1)} \cdot \left(\frac{1}{x^2+1} \cdot \ln(2x+1) + \arctan(x) \cdot \frac{1}{2x+1} \cdot 2\right)$$

$$\frac{dy}{dx} = (2x+1)^{\arctan(x)}\left(\frac{\ln(2x+1)}{x^2+1} + \frac{2\arctan(x)}{2x+1}\right)$$