Math 220 (section AD?) Quiz 4 Fall 2016

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) What is the slope of the curve $y = 10 - 2e^x$ at its x-intercept? Simplify your answer.

To find x-intercept, set $y=0$

$0 = 10 - 2e^x$

$2e^x = 10$

$e^x = 5$

$\ln(e^x) = \ln(5)$

$x = \ln(5)$

$y' = 0 - 2e^x$

$y'(\ln(5)) = -2e^{\ln(5)}$

$= -2 \cdot 5$

$= -10$

2. (2 points) Determine the equation of the line which is tangent to the curve $f(x) = x^4 + 6x + 10$ and perpendicular to the line $x + 2y = 16$.

(1) $x + 2y = 16$

$2y = -x + 16$

$y = -\frac{1}{2}x + 8$

has slope $-\frac{1}{2}$

a line perpendicular to this line has slope $-\frac{1}{(-1/2)} = 2$

(2) The slope of each line tangent to $f(x)$ is given by $f'(x) = 4x^3 + 6$

(3) set $f'(x) = 2$

$4x^3 + 6 = 2$

$4x^3 = -4$

$x^3 = -1$

$x = -1$

(4) Point: $(-1, f(-1)) = (-1, 5)$

slope: 2

tangent line: $y - 5 = 2(x - (-1)) \Rightarrow \boxed{y = 2x + 7}$

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $q = \left(\frac{-2x^3}{\sqrt[3]{2x}}\right)^3 + \cos(2\arcsin(3/5))$ (simplify your answer)

$$q = \frac{(-2x^3)^3}{(\sqrt[3]{2x})^3} + \text{constant}$$

$$q = \frac{(-2)^3(x^3)^3}{2x} + \text{constant}$$

$$q = \frac{-8x^9}{2x} + \text{constant}$$

$$q = -4x^8 + \text{constant} \Rightarrow \frac{dq}{dx} = -32x^7$$

(b) $p = v^6 \sec(v)$

$$\frac{dp}{dv} = \frac{d}{dv}(v^6)(\sec(v)) + (v^6)\frac{d}{dv}(\sec(v))$$

$$\frac{dp}{dv} = 6v^5\sec(v) + v^6\sec(v)\tan(v)$$

(c) $r = \frac{5 + 2\sin t}{t^4}$

$$\frac{dr}{dt} = \frac{\frac{d}{dt}(5+2\sin(t))\cdot t^4 - (5+2\sin(t))\cdot\frac{d}{dt}(t^4)}{(t^4)^2}$$

$$\frac{dr}{dt} = \frac{2\cos(t)\cdot t^4 - (5+2\sin(t))\cdot 4t^3}{t^8}$$