Name solutions

• 20 minutes • No calculators • Show sufficient work • Do not use derivatives

1. (2 points) Evaluate $\cos(2\arctan(2/3))$. $= \cos(2\theta)$

Let $\theta = \arctan(\frac{2}{3})$

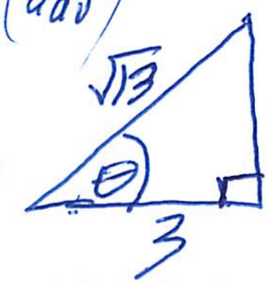
Then $\tan(\theta) = \frac{2}{3}$ $\left(\frac{\text{opp}}{\text{adj}}\right)$

and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

we further have

$0 < \theta < \frac{\pi}{2}$

since $\tan(\theta) > 0$



$2^2 + 3^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{13}$

$= \cos^2(\theta) - \sin^2(\theta)$

$= \left(\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$

$= \frac{9}{13} - \frac{4}{13}$

$= \frac{5}{13}$

2. (2 points each) Evaluate the following limits. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\lim_{x\to\infty} \frac{8e^x + 5}{3 + 4e^x}$ (numerator $\to \infty$, denominator $\to \infty$)

$= \lim_{x\to\infty} \frac{8e^x + 5}{3 + 4e^x} \cdot \frac{1/e^x}{1/e^x}$

$= \lim_{x\to\infty} \frac{8 + 5/e^x}{3/e^x + 4}$ (numerator $\to 8$, denominator $\to 4$)

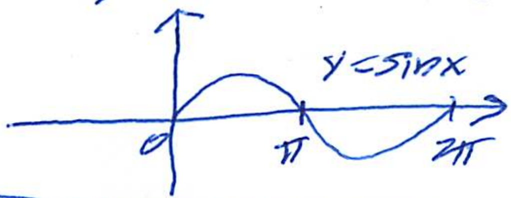
$= \frac{8}{4}$

$= 2$

(b) $\lim_{x\to\pi^+} \frac{x^2}{\sin x}$ $\to \pi^2$, $\to 0^-$

$= -\infty$

Note: From graph, $\sin(x) \to 0^-$ as $x \to \pi^+$



(c) $\lim_{x\to 4} \frac{x-4}{5-\sqrt{3x+13}}$ $\to 0$, $\to 0$

$$= \lim_{x\to 4} \frac{x-4}{5-\sqrt{3x+13}} \cdot \frac{5+\sqrt{3x+13}}{5+\sqrt{3x+13}}$$

$$= \lim_{x\to 4} \frac{(x-4)(5+\sqrt{3x+13})}{(5)^2-(\sqrt{3x+13})^2}$$

$$= \lim_{x\to 4} \frac{(x-4)(5+\sqrt{3x+13})}{25-(3x+13)}$$

$$= \lim_{x\to 4} \frac{(x-4)(5+\sqrt{3x+13})}{-3x+12}$$

$$= \lim_{x\to 4} \frac{(x-4)(5+\sqrt{3x+13})}{-3(x-4)}$$

$$= \lim_{x\to 4} \frac{5+\sqrt{3x+13}}{-3}$$

$$= \frac{5+\sqrt{3\cdot 4+13}}{-3}$$

$$= \frac{-10}{3}$$

3. (2 points) Determine an equation for each vertical asymptote on the graph of the following function. Your answer must be justified using limits.

$$f(x) = \frac{x^2 + x - 6}{2x^2 - 8}$$

$$= \frac{x^2+x-6}{2(x^2-4)}$$

$$= \frac{x^2+x-6}{2(x-2)(x+2)}$$

possible asymptotes at $x=2$ or $x=-2$

check $\lim_{x\to 2} \frac{x^2+x-6 \to 0}{2x^2-8 \to 0} = \lim_{x\to 2} \frac{(x+3)(x-2)}{2(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{x+3 \to 5}{2(x+2) \to 8}$

$= \frac{5}{8}$ no vertical asymptote at $x=2$

check $\lim_{x\to -2^+} \frac{x^2+x-6}{2x^2-8} = \lim_{x\to -2^+} \frac{x+3 \to 1}{2(x+2) \to 0^+}$

$= \infty$

f has only one vertical asymptote. It is at $x=-2$

(or could check $\lim_{x\to -2^-} \frac{x+3 \to 1}{2(x+2) \to 0^-} = -\infty$)