Math 220 (section AD?) Quiz 2 Fall 2016

Name Solutions

- You have 18 minutes
- No calculators
- Show sufficient work

1. (3 points) Determine all values of x which satisfy the equation below.

$$25 - 6x = e^{2\ln(3-x)}$$

$25-6x = e^{\ln((3-x)^2)}$ (we used the properties $r\cdot\ln(u)=\ln(u^r)$ and $e^{\ln(w)}=w$)

$25-6x = (3-x)^2$

$25-6x = 9-6x+x^2$

$16 = x^2$

$x = \pm 4$

however, 4 is not in the domain of $e^{2\ln(3-x)}$ since $3-4=-1$ and $\ln(-1)$ is not a real number.

Thus, $x=-4$ is the only solution

2. (2 points) If $h(x) = \dfrac{2}{3+5e^{4x}}$ then determine a formula for $h^{-1}(x)$.

$y = \dfrac{2}{3+5e^{4x}}$

$x = \dfrac{2}{3+5e^{4y}}$ (switch x & y)

$x(3+5e^{4y}) = 2$ (solve for y)

$3+5e^{4y} = \dfrac{2}{x}$

$5e^{4y} = \dfrac{2}{x} - 3$

$e^{4y} = \dfrac{\frac{2}{x}-3}{5}$

$\ln(e^{4y}) = \ln\left(\dfrac{\frac{2}{x}-3}{5}\right)$

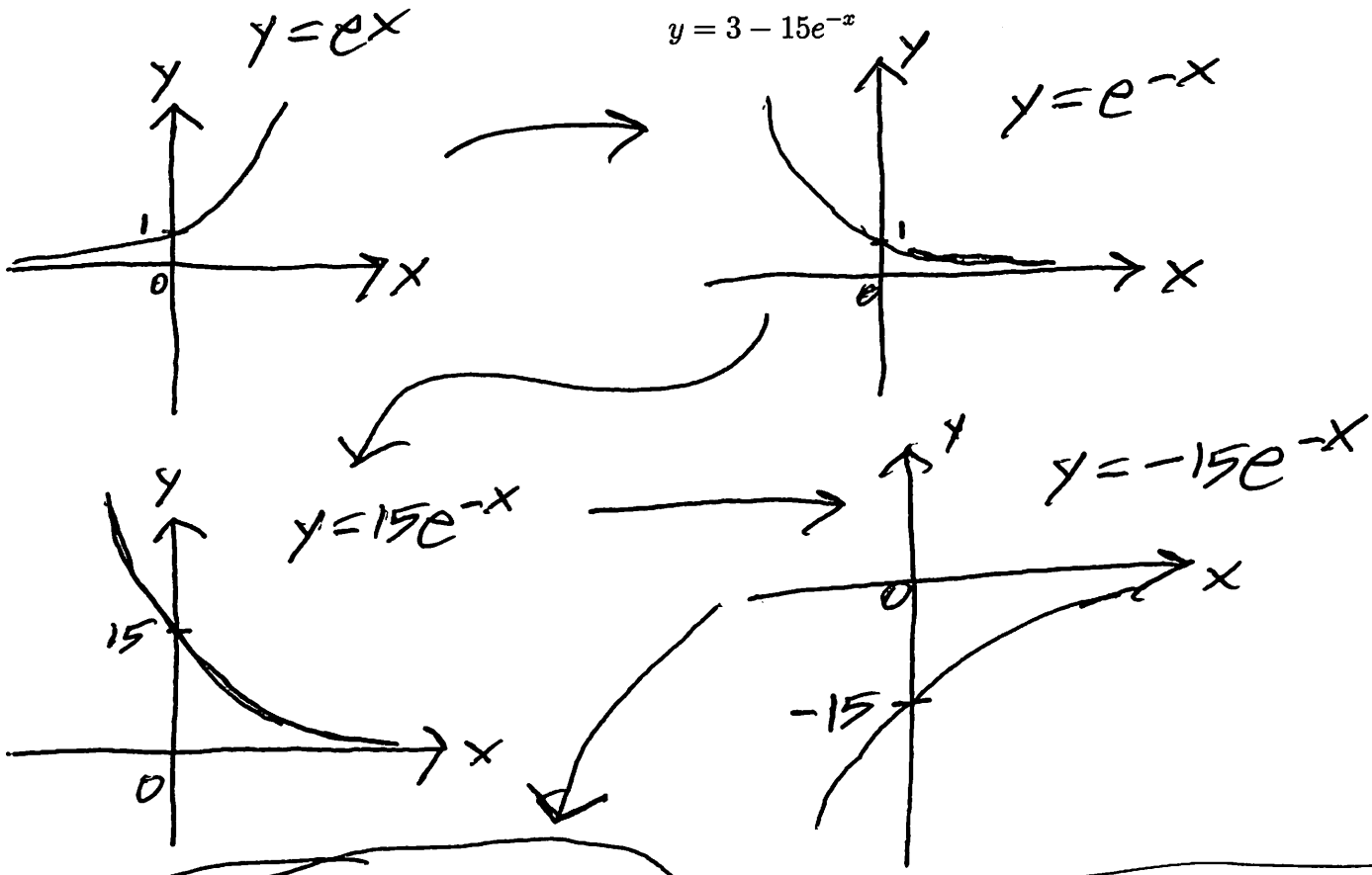
$4y = \ln\left(\dfrac{\frac{2}{x}-3}{5}\right)$

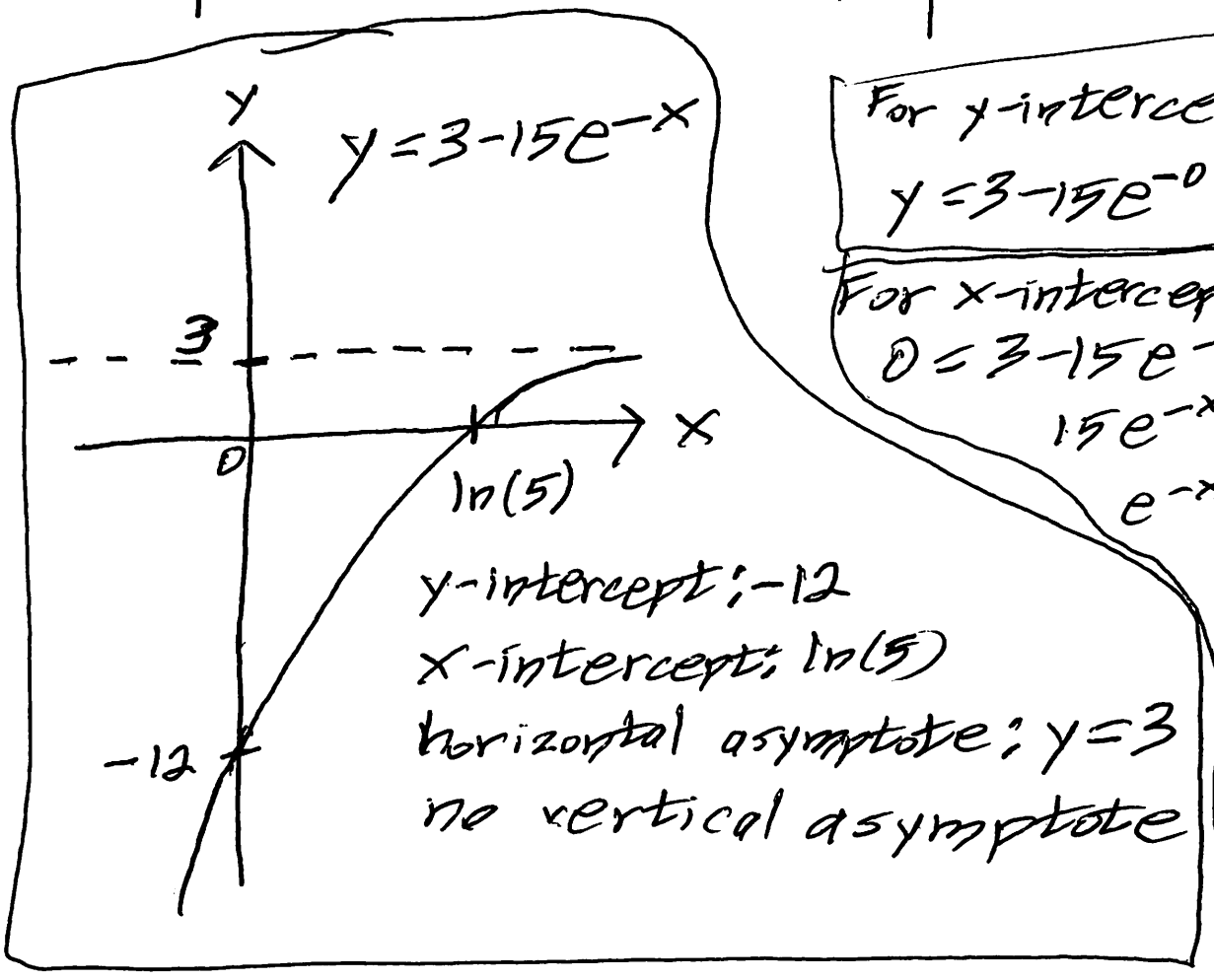
$y = \dfrac{1}{4}\ln\left(\dfrac{\frac{2}{x}-3}{5}\right)$

$h^{-1}(x) = \dfrac{1}{4}\ln\left(\dfrac{\frac{2}{x}-3}{5}\right)$

3. (3 points) Carefully sketch a graph of the following function. You should clearly label the value of any x-intercepts, y-intercepts, or asymptotes.

$y = 3 - 15e^{-x}$





y-intercept: -12

x-intercept: $\ln(5)$

horizontal asymptote: $y = 3$

no vertical asymptote

For y-intercept, set $x = 0$

$y = 3 - 15e^{0} = -12$

For x-intercept, set $y = 0$

$0 = 3 - 15e^{-x}$

$15e^{-x} = 3$

$e^{-x} = \frac{3}{15} = \frac{1}{5}$

$\ln(e^{-x}) = \ln(\frac{1}{5})$

$-x = \ln(1) - \ln(5)$

$-x = 0 - \ln(5)$

$x = \ln(5)$

4. (2 points) Determine a formula for an exponential function given that its graph goes through the points $(1, 10)$, $(5, 30)$ and $(9, 90)$.

$y = C \cdot a^x$

$(1,10)$ on graph $\Rightarrow 10 = C \cdot a^1 \Rightarrow C = \frac{10}{a}$

$(5,30)$ on graph $\Rightarrow 30 = C \cdot a^5$

$\Rightarrow 30 = \left(\frac{10}{a}\right) \cdot a^5$

$\Rightarrow 3 = a^4$

$\Rightarrow a = \sqrt[4]{3} = 3^{1/4}$

$C = \frac{10}{3^{1/4}}$

$y = \frac{10}{3^{1/4}} \cdot \left(3^{1/4}\right)^x$

$y = \frac{10}{3^{1/4}} \cdot 3^{x/4}$

note $(9,90)$ is on the graph of this function but it was not needed to find the formula.