Math 220 **Quiz 12 (take-home)** **Fall 2016**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Andrew McConvey
- ▷ **AD2**, TR 9:00-10:50, Ben Wright
- ▷ **AD3**, TR 1:00-2:50, Cassie Christenson
- ▷ **ADA**, TR 8:00-8:50, Alexi Block Gorman
- ▷ **ADB**, TR 9:00-9:50, Dakota Ihli
- ▷ **ADC**, TR 10:00-10:50, Elizabeth Field
- ▷ **ADD**, TR 11:00-11:50, Adam Wagner
- ▷ **ADE**, TR 12:00-12:50, Adam Wagner
- ▷ **ADF**, TR 1:00-1:50, Tsutomu Okano
- ▷ **ADG**, TR 2:00-2:50, Xujun 'Henry' Liu
- ▷ **ADH**, TR 3:00-3:50, Mychael Sanchez
- ▷ **ADI**, TR 4:00-4:50, Mychael Sanchez
- ▷ **ADJ**, TR 9:00-9:50, Kyle Pratt
- ▷ **ADK**, TR 10:00-10:50, Kyle Pratt
- ▷ **ADL**, TR 11:00-11:50, Tigran Hakobyan
- ▷ **ADM**, TR 12:00-12:50, Liz Tatum
- ▷ **ADN**, TR 1:00-1:50, Xujun 'Henry' Liu
- ▷ **ADO**, TR 2:00-2:50, Tigran Hakobyan
- ▷ **ADP**, TR 3:00-3:50, Liz Tatum
- ▷ **ADQ**, TR 10:00-10:50, Dakota Ihli
- ▷ **ADR**, TR 9:00-9:50, Elizabeth Field
- ▷ **ADS**, TR 12:00-12:50, Tsutomu Okano
- ▷ **ADT**, TR 2:00-2:50, Anna Weigandt
- ▷ **ADU**, TR 3:00-3:50, Anna Weigandt

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official lecture period on Friday, November 18.**
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until the quizzes have been collected for all of my lectures (9am, noon, 1pm).**

1. (3 points) A calculator gives an estimate of 5.098064112 for the value of $\sqrt[3]{132.5}$.

   Using the techniques of linear approximation found in section 3.10, show that you are able to obtain a very similar estimate of 5.1 without the use of any technology.

Find Tangent line to $f(x) = \sqrt[3]{x}$ at $x = 125$

point: $(125, f(125)) = (125, \sqrt[3]{125}) = (125, 5)$

slope: $f'(x) = \frac{1}{3}x^{-2/3} \Rightarrow f'(125) = \frac{1}{3}(125)^{-2/3}$
$= \frac{1}{75}$

tangent line: $y - 5 = \frac{1}{75}(x - 125)$

$y = 5 + \frac{1}{75}(x - 125)$

$L(x) = 5 + \frac{1}{75}(x - 125)$

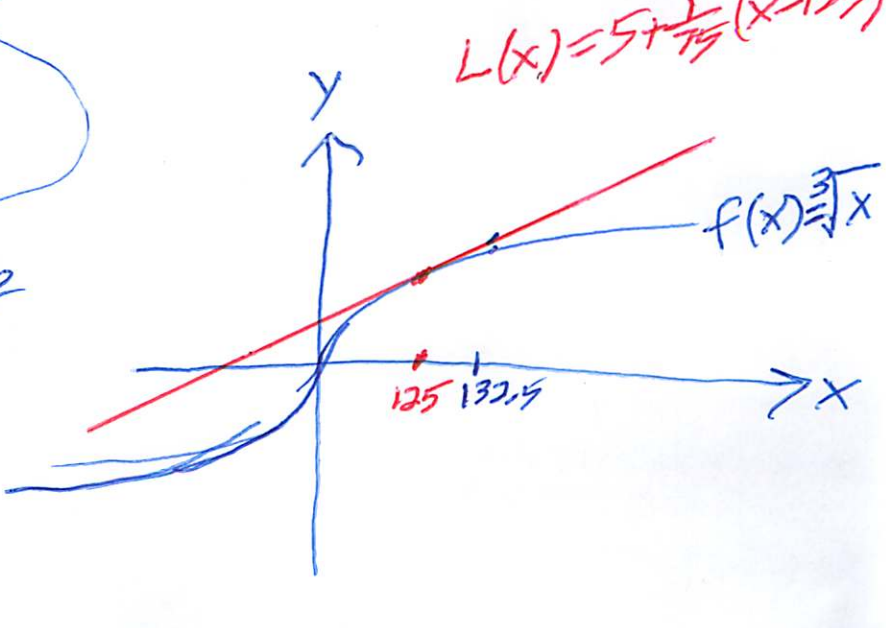
$f(x) \approx L(x)$ for x near the point of tangency

$\sqrt[3]{x} \approx 5 + \frac{1}{75}(x - 125)$ for x near 125

$\sqrt[3]{132.5} \approx 5 + \frac{1}{75}(132.5 - 125)$

$\sqrt[3]{132.5} \approx 5.1$

From the graph we see this is an overestimate



2. (3 points) Suppose that f and f' are differentiable everywhere and the following conditions hold.

- $f(5) = 8$
- $f'(5) = 3$
- $f''(5) = 2$

Use the techniques of linear approximation found in section 3.10 to estimate the following quantities. Simplify and write your answers in decimal form.

(a) $f(4.8)$

on graph of $f(x)$ we have

point: $(5, f(5)) = (5, 8)$

slope: $f'(5) = 3$

tang. line: $y - 8 = 3(x-5)$

$y = 8 + 3(x-5)$

thus, $f(x) \approx 8 + 3(x-5)$ for x near 5

$f(4.8) \approx 8 + 3(4.8 - 5)$

$f(4.8) \approx 7.4$

(b) $f'(4.8)$

on graph of $f'(x)$ we have

point: $(5, f'(5)) = (5, 3)$

slope: $f''(5) = 2$

tang. line: $y - 3 = 2(x-5)$

$y = 3 + 2(x-5)$

thus, $f'(x) \approx 3 + 2(x-5)$ for x near 5

$f'(4.8) \approx 3 + 2(4.8 - 5)$

$f'(4.8) \approx 2.6$

3. (4 points) Estimate the x-intercept on the graph of $f(x) = 5x^5 - x^2 - 24$ using Newton's Method with an initial estimate of $x_1 = 2$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

Since we are estimating a solution to $f(x)=0$, we can apply Newton's Method directly.

$$f(x)=5x^5-x^2-24 \Rightarrow f'(x)=25x^4-2x$$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=x_n-\frac{5x_n^5-x_n^2-24}{25x_n^4-2x_n}$$

$x_1 = 2$ given as first estimate

$$x_2=x_1-\frac{5x_1^5-x_1^2-24}{25x_1^4-2x_1}=2-\frac{5(2)^5-(2)^2-24}{25(2)^4-2(2)}\approx 1.666666667$$

$$x_3=x_2-\frac{5x_2^5-x_2^2-24}{25x_2^4-2x_2}$$

$$x_3\approx 1.666666667-\frac{5(1.666666667)^5-(1.666666667)^2-24}{25(1.666666667)^4-2(1.666666667)}$$

$x_3 \approx 1.468728970$

$$x_4=x_3-\frac{5x_3^5-x_3^2-24}{25x_3^4-2x_3}$$

$$x_4\approx 1.468728970-\frac{5(1.468728970)^5-(1.468728970)^2-24}{25(1.468728970)^4-2(1.468728970)}$$

$x_4 \approx 1.398043764$