Math 220 (section AD?) **Quiz 11** **Fall 2016**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{30x^2}{x^3+12}$ on the interval $[1, 3]$. Simplify your answer.

$$f_{ave} = \frac{1}{3-1}\int_1^3 \frac{30x^2}{x^3+12}\,dx$$

$$= \frac{1}{2}\int_1^3 \frac{10}{x^3+12}\cdot 3x^2\,dx$$

$$= \frac{1}{2}\int_{13}^{39} \frac{10}{u}\,du$$

$$= 5\Big[\ln|u|\Big]_{13}^{39}$$

$$= 5\left(\ln(39) - \ln(13)\right)$$

$$= 5\ln\left(\frac{39}{13}\right)$$

$$= 5\ln(3)$$

u-substitution

$u = x^3 + 12$

$du = 3x^2\,dx$

$x = 1 \Rightarrow u = 1^3 + 12 = 13$

$x = 3 \Rightarrow u = 3^3 + 12 = 39$

2. Let **R** be the finite region bounded by the graphs of $x = 0$, $y = 3e^{2x}$ and $y = 24e^{-x}$. Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are semi-circles.

intersection

$3e^{2x} = 24e^{-x}$

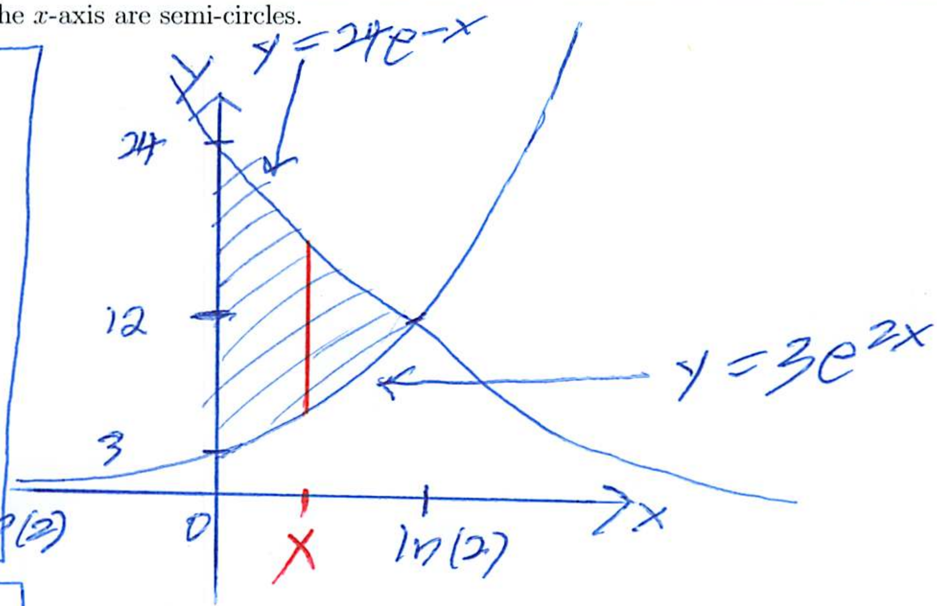
$e^{2x} = \frac{8}{e^{x}}$

$e^{2x} \cdot e^{x} = 8$

$e^{3x} = 8$

$\ln(e^{3x}) = \ln(8)$

$3x = \ln(8) \Rightarrow x = \frac{\ln(8)}{3} = \ln(2)$



cross-section at x is a semi-circle

$$V = \int_0^{\ln(2)} \begin{pmatrix} \text{cross-sectional} \\ \text{area} \end{pmatrix} dx$$

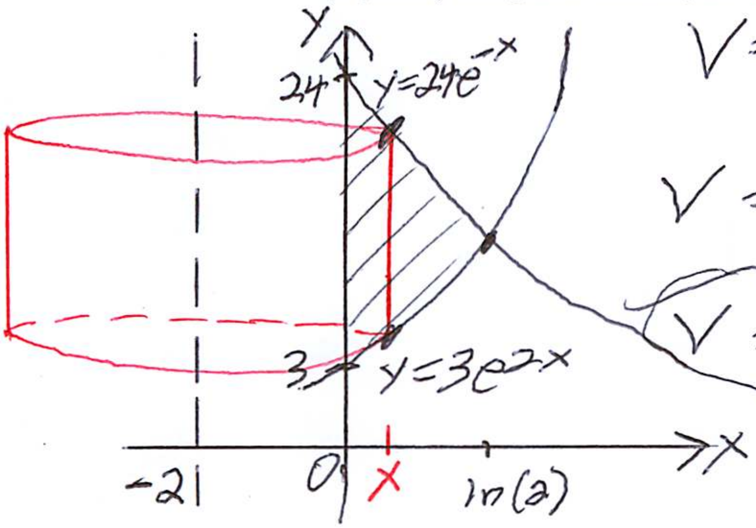
$$V = \int_0^{\ln(2)} \frac{1}{2}\pi r^2 \, dx$$

$$V = \int_0^{\ln(2)} \frac{1}{2}\pi \left(\frac{1}{2}(\text{diameter})\right)^2 dx$$

$$V = \int_0^{\ln(2)} \frac{1}{2}\pi \left(\frac{1}{2}(24e^{-x} - 3e^{2x})\right)^2 dx$$

(b) The volume of the solid formed when $\mathbf{R}$ is revolved around the line $x = -2$. Determine this volume in the following two ways.

i. (2 points) Integrate with respect to x.

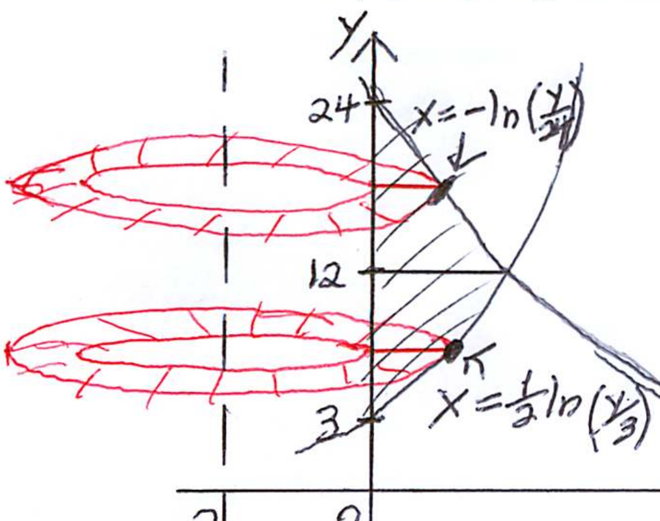


$$V = \int_{x_{min}}^{x_{max}} \left(\begin{array}{c}\text{surface}\\ \text{area}\end{array}\right) dx$$

$$V = \int_0^{\ln(2)} 2\pi r h \, dx$$

$$V = \int_0^{\ln(2)} 2\pi (x-(-2))(24e^{-x} - 3e^{2x}) \, dx$$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)



$$y = 3e^{2x} \Rightarrow x = \tfrac{1}{2}\ln\left(\tfrac{y}{3}\right)$$

$$y = 24e^{-x} \Rightarrow x = -\ln\left(\tfrac{y}{24}\right)$$

$$V = \int_{y_{min}}^{y_{max}} \left(\begin{array}{c}\text{cross-sectional}\\ \text{area}\end{array}\right) dy$$

$$V = \int_3^{12} \left(\pi\left(\tfrac{1}{2}\ln\left(\tfrac{y}{3}\right) - (-2)\right)^2 - \pi(0-(-2))^2\right) dy$$

$$+$$

$$\int_{12}^{24} \left(\pi\left(-\ln\left(\tfrac{y}{24}\right) - (-2)\right)^2 - \pi(0-(-2))^2\right) dy$$

cross-section at y is