Math 220 (section AD?) **Quiz 10** **Fall 2016**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

suppose the following conditions hold.

① f is continuous on [a, b]

② f is differentiable on (a, b)

Then there is a c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_{e^4}^{e^9} \frac{6\sqrt{\ln x}}{x}\, dx = \int_{e^4}^{e^9} 6\sqrt{\ln(x)} \cdot \frac{1}{x}\, dx$$

Let $u = \ln(x)$

$du = \frac{1}{x} dx$

limits of integration

$x = e^4 \Rightarrow u = \ln(e^4) = 4$

$x = e^9 \Rightarrow u = \ln(e^9) = 9$

$$= \int_4^9 6\sqrt{u}\, du$$

$$= \left[6 \cdot \frac{1}{3/2} u^{3/2}\right]_4^9$$

$$= 4(9)^{3/2} - 4(4)^{3/2}$$

$$= 4 \cdot 27 - 4 \cdot 8$$

$$= 108 - 32$$

$$= 76$$

3. (2 points) Evaluate the indefinite integral.

$$\int 15x^4 \sin^2\left(x^5\right) \cos\left(x^5\right)\, dx = \int 3 \sin^2(x^5)\cos(x^5)\cdot 5x^4\,dx$$

$$\boxed{\begin{array}{l} u = x^5 \\ du = 5x^4\,dx \end{array}}$$

$$= \int 3\sin^2(u)\cos(u)\,du$$

$$\boxed{\begin{array}{l} w = \sin(u) \\ dw = \cos(u)\,du \end{array}}$$

$$= \int 3w^2\,dw$$

$$= w^3 + C$$

$$= \sin^3(u) + C$$

$$= \sin^3(x^5) + C$$

<u>another solution</u>

$$\int 15x^4 \sin^2(x^5)\cos(x^5)\,dx = \int 3u^2\,du$$

$$\boxed{\begin{array}{l} u = \sin(x^5) \\ du = 5x^4\cos(x^5)\,dx \end{array}}$$

$$= u^3 + C$$

$$= \sin^3(x^5) + C$$

note: $e^{2x} = (e^x)^2$

4. (2 points) Evaluate the indefinite integral.

$$\int e^{2x}\left(e^x+3\right)^{10}\,dx$$

$$= \int e^x (e^x+3)^{10} e^x\,dx$$

$$\boxed{\begin{aligned} u &= e^x+3 \\ du &= e^x\,dx \end{aligned}}$$

$\rightarrow e^x = u-3$

$$= \int (u-3)\,u^{10}\,du$$

$$= \int (u^{11} - 3u^{10})\,du$$

$$= \frac{1}{12}u^{12} - 3\cdot\frac{1}{11}u^{11} + C$$

$$= \frac{1}{12}(e^x+3)^{12} - \frac{3}{11}(e^x+3)^{11} + C$$

5. (2 points) Let **R** be the finite region bounded by the given functions. In the following way, set up but do not evaluate definite integrals which represent the area of the region **R**.

$$f(x) = -2x + 14$$

$$g(x) = \frac{20}{x}$$

$\frac{20}{x} = -2x + 14$

$20 = -2x^2 + 14x$

$2x^2 - 14x + 20 = 0$

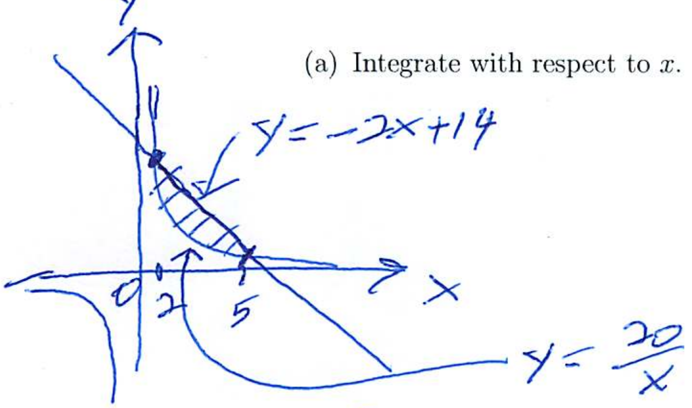
$2(x^2 - 7x + 10) = 0$

$2(x-2)(x-5) = 0$

$x = 2, \; x = 5$

intersection points are $(2, f(2)) = (2, 10)$ and $(5, f(5)) = (5, 4)$

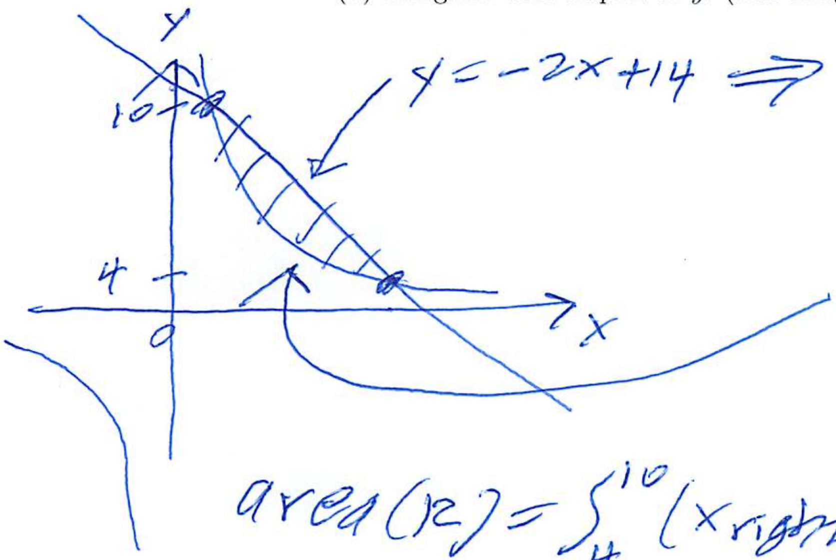
(a) Integrate with respect to x.



$\text{area}(R) = \int_2^5 (y_{top} - y_{bottom})\,dx$

$\text{area}(R) = \int_2^5 \left((-2x+14) - \left(\frac{20}{x}\right)\right) dx$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)



$y = -2x + 14 \Rightarrow x = \frac{14-y}{2}$

$y = \frac{20}{x} \Rightarrow x = \frac{20}{y}$

$\text{area}(R) = \int_4^{10} (x_{right} - x_{left})\,dy$

$\text{area}(R) = \int_4^{10} \left(\frac{14-y}{2} - \frac{20}{y}\right) dy$