Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Given an acute angle θ for which $\sin(\theta) = \frac{1}{3}$, evaluate the following quantities.

(a) $\sec(\theta)$

Method 1

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos^2(\theta) + (\frac{1}{3})^2 = 1$

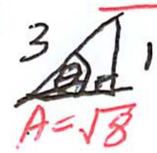
$\cos^2(\theta) = 1 - \frac{1}{9} = \frac{8}{9}$

$\cos(\theta) = \pm\sqrt{\frac{8}{9}}$

θ acute $\Rightarrow \cos(\theta) = \sqrt{8/9}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{\sqrt{8/9}} = \frac{3\sqrt{2}}{4}$

Method 2



$\sin(\theta) = \frac{1}{3}$ $\left(\frac{opp}{hyp}\right)$

Pythagorean Theorem $\Rightarrow$

$A^2 + 1^2 = 3^2 \Rightarrow A = \sqrt{8}$

$\sec(\theta) = \frac{3}{\sqrt{8}}$ $\left(\frac{hyp}{adj}\right)$

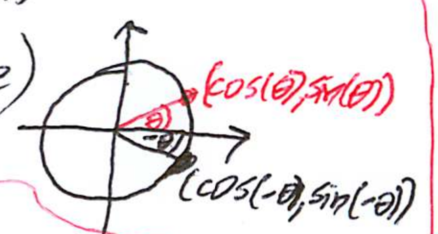
$= \frac{3\sqrt{2}}{4}$

(b) $\sin(-\theta)$

$\sin(-\theta) = -\sin(\theta)$ since sine is an odd function

$= -\frac{1}{3}$

(see unit circle)



(c) $\cos(\pi - \theta)$

Method 1

unit circle

$\cos(\pi - \theta) = -\cos(\theta)$

$= -\sqrt{\frac{8}{9}}$

$= -\frac{2\sqrt{2}}{3}$

Method 2

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\cos(\pi - \theta) = \cos(\pi)\cos(\theta) + \sin(\pi)\sin(\theta)$

$= -1 \cdot \cos(\theta) + 0 \cdot \sin(\theta)$

$= -\cos(\theta)$

$= -\frac{2\sqrt{2}}{3}$

2. (4 points) Determine the domain of the given function.

$$f(x) = \frac{\sin(x^2 - 4)}{\sqrt{102 - \sqrt{200 - 2x^2}}}$$

No restrictions on domain from this numerator

From $\sqrt{200-2x^2}$ we get

$200-2x^2 \geq 0 \Rightarrow -2x^2 \geq -200$

$\Rightarrow x^2 \leq 100$

$\Rightarrow \sqrt{x^2} \leq \sqrt{100}$

$\Rightarrow |x| \leq 10$

$\Rightarrow -10 \leq x \leq 10$

The denominator equals 0 when

$\sqrt{102} - \sqrt{200-2x^2} = 0$

$\sqrt{102} = \sqrt{200-2x^2}$

$102 = 200 - 2x^2$

$2x^2 = 98$

$x^2 = 49$

$x = \pm 7$ Thus $x \neq \pm 7$

Domain of $f(x)$ is

$[-10,-7) \cup (-7,7) \cup (7,10]$

3. (3 points) Determine whether the following function is even, odd or neither. Give a very clear justification for your answer.

$$w(t) = t^5 \sin\left(t^3\right)$$

$w(-t) = (-t)^5 \sin((-t)^3)$

$= -t^5 \sin(-t^3)$

$= -t^5 \cdot -\sin(t^3)$ since sine is an odd function

$= t^5 \sin(t^3)$

$= w(t)$

$w(-t) = w(t) \Rightarrow w$ is an even function