Math 220 (section AD?) Quiz 9 Fall 2015

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_{-3}^{-1} \frac{x-6}{x^2}\,dx = \int_{-3}^{-1}\left(\frac{x}{x^2}-\frac{6}{x^2}\right)dx$$
$$= \int_{-3}^{-1}\left(\frac{1}{x}-6x^{-2}\right)dx$$
$$= \left[\ln|x| + 6x^{-1}\right]_{-3}^{-1}$$
$$= \left(\ln|-1| + \frac{6}{-1}\right) - \left(\ln|-3| + \frac{6}{-3}\right)$$
$$= (0-6) - (\ln(3)-2) = -4-\ln(3)$$

2. (2 points each) Evaluate the following indefinite integrals.

(a) $$\int \frac{5x^2-5}{x^4-1}\,dx = \int \frac{5(x^2-1)}{(x^2+1)(x^2-1)}\,dx$$
$$= 5\int \frac{1}{x^2+1}\,dx$$
$$= 5\arctan(x) + C$$

(b) $$\int \frac{\sin(x)\cos^2(x)}{(1-\sin^2(x))^2}\,dx = \int \frac{\sin(x)\cos^2(x)}{(\cos^2(x))^2}\,dx$$
$$= \int \frac{\sin(x)}{\cos^2(x)}\,dx = \int \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)}\,dx$$
$$= \int \sec(x)\tan(x)\,dx$$
$$= \sec(x) + C$$

3. (2 points) Suppose $g(x) = \int_2^{x^5} \sin(t^2)\, dt$. Find $g''(x)$.

$g'(x) = \sin((x^5)^2) \cdot \frac{d}{dx}(x^5)$ By F.T.C. part I and chain rule

$= \sin(x^{10}) \cdot 5x^4$

$g''(x) = \frac{d}{dx}(\sin(x^{10})) \cdot 5x^4 + \sin(x^{10}) \cdot \frac{d}{dx}(5x^4)$

$= \cos(x^{10}) \cdot 10x^9 \cdot 5x^4 + \sin(x^{10}) \cdot 20x^3$

$= 50x^{13}\cos(x^{10}) + 20x^3\sin(x^{10})$

4. (2 points) The height of a tree is currently 20 inches. If the tree's height increases by $6t$ inches per year where t is measured in years from now, then what will the tree's height be in 5 years?

$\begin{pmatrix}\text{Tree's height} \\ \text{at } t=5\end{pmatrix} = \begin{pmatrix}\text{Tree's height} \\ \text{at } t=0\end{pmatrix} + \begin{pmatrix}\text{Tree's net change} \\ \text{in height from} \\ t=0 \text{ to } t=5\end{pmatrix}$

$= 20 + \int_0^5 6t\, dt$

$= 20 + \left[3t^2\right]_0^5$

$= 20 + (3 \cdot 5^2 - 3 \cdot 0^2)$

$= 20 + 75$

$= 95$ inches