Name: Solutions

(circle your TA discussion section)

- AD1, TR 11:00-12:50, Derek Jung
- AD2, TR 9:00-10:50, Claire Merriman
- AD3, TR 1:00-2:50, Itziar Ochoa de Alaiza Gracia
- ADA, TR 8:00-8:50, Dara Zirlin
- ADB, TR 9:00-9:50, Dara Zirlin
- ADC, TR 10:00-10:50, Xujun Liu
- ADD, TR 11:00-11:50, Christopher Linden
- ADE, TR 12:00-12:50, Christopher Linden
- ADF, TR 1:00-1:50, Alyssa Loving
- ADG, TR 2:00-2:50, Xianchung Meng
- ADH, TR 3:00-3:50, Xianchung Meng
- ADI, TR 4:00-4:50, Aaron Schneberger

- ADJ, TR 9:00-9:50, Elizabeth Field
- ADK, TR 10:00-10:50, Elizabeth Field
- ADL, TR 11:00-11:50, Emily Heath
- ADM, TR 12:00-12:50, Alyssa Loving
- ADN, TR 1:00-1:50, Aaron Schneberger
- ADO, TR 2:00-2:50, Tigran Hakobyan
- ADP, TR 3:00-3:50, Tigran Hakobyan
- ADR, TR 9:00-9:50, Xujun Liu
- ADS, TR 12:00-12:50, Emily Heath
- ADT, TR 2:00-2:50, Argen West
- ADU, TR 3:00-3:50, Argen West

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.

- You may use your notes, the textbook, or information found on my course home page.

- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.

- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.

- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.

- Be sure that the pages are nicely stapled – do not just fold the corners.

- The quiz is due at the beginning of your official discussion period on Tues., Nov. 3rd.

- Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Tuesday.
1. (2 points) Find a formula for $g(t)$ given that $g''(t) = 8 \cos(t) - 2 \sin(t)$, $g(0) = -2$ and $g(\pi/2) = \pi - 3$.

\[
g''(t) = 8 \cos(t) - 2 \sin(t) \Rightarrow \\
g'(t) = 8 \sin(t) + 2 \cos(t) + C_1 \Rightarrow \\
g(t) = -8 \cos(t) + 2 \sin(t) + C_1 t + C_2 \]

$g(0) = -2 \Rightarrow$

\[-8 \cos(0) + 2 \sin(0) + C_1(0) + C_2 = -2 \]

$-8 + C_2 = -2 \Rightarrow C_2 = 6$

$g(\pi/2) = \pi - 3 \Rightarrow$

\[-8 \cos(\pi/2) + 2 \sin(\pi/2) + C_1(\pi/2) + 6 = \pi - 3 \]

$2 + C_1(\pi/2) + 6 = \pi - 3 \Rightarrow C_1 = \frac{2}{\pi} (\pi - 11) = \frac{2\pi - 22}{\pi}$

$g(t) = -8 \cos(t) + 2 \sin(t) + \frac{2\pi - 22}{\pi} t + 6$
2. (2 points) Suppose that \( w(x) \) is continuous at all real numbers and satisfies the following equations.

\[
\begin{align*}
\int_6^{12} w(x) \, dx &= 5 \\
\int_6^{4} w(x) \, dx &= 15 \\
\int_1^{12} w(x) \, dx &= 25
\end{align*}
\]

we use properties

\[
\begin{align*}
\int_a^b w(x) \, dx &= -\int_a^b w(x) \, dx \\
\int_a^b w(x) \, dx &= \int_a^c w(x) \, dx + \int_c^b w(x) \, dx
\end{align*}
\]

and

\[
\int_a^b w(x) \, dx = \int_a^c w(x) \, dx + \int_c^b w(x) \, dx
\]

What is the value of \( \int_1^4 (2w(x) - 50) \, dx \)?

\[
\begin{align*}
\int_1^{12} w(x) \, dx &= \int_1^{12} w(x) \, dx + \int_1^{4} w(x) \, dx \\
&= \int_1^{12} w(x) \, dx + (\int_1^{12} w(x) \, dx + \int_6^{12} w(x) \, dx) \\
&= \int_1^{12} w(x) \, dx + (\int_6^{12} w(x) \, dx + \int_6^{12} w(x) \, dx) \\
&= 25 + (-5 + 15) \\
&= 35
\end{align*}
\]

\[
\int_1^{12} (6w(x) - 50) \, dx = 2 \int_1^{12} w(x) \, dx - \int_1^{12} 50 \, dx
\]

\[
= 2 \cdot 35 - [50x]_1^4
\]

\[
= 70 - [50 \cdot 4 - 50 \cdot 1]
\]

\[
= -80
\]
3. (2 points) Evaluate the following limit. Use proper notation throughout your evaluation of this limit.

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2n^2 - 10kn + 24k^2 - 42}{n^3} = \\
\lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{2n^2}{n^3} - \frac{10kn}{n^3} + \frac{24k^2}{n^3} - \frac{42}{n^3} \right) = \\
\lim_{n \to \infty} \left( \frac{\sum_{k=1}^{n} 2n^2}{n^3} - \frac{\sum_{k=1}^{n} 10kn}{n^3} + \frac{\sum_{k=1}^{n} 24k^2}{n^3} - \frac{\sum_{k=1}^{n} 42}{n^3} \right) = \\
\lim_{n \to \infty} \left( \frac{2}{n^2} \sum_{k=1}^{n} 1 - \frac{10}{n^2} \sum_{k=1}^{n} k + \frac{24}{n^3} \sum_{k=1}^{n} k^2 - \frac{42}{n^3} \sum_{k=1}^{n} 1 \right) = \\
\lim_{n \to \infty} \left( \frac{2}{n^2} \cdot n - \frac{10}{n^2} \cdot \frac{n(n+1)}{2} + \frac{24}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{42}{n^3} \cdot n \right) = \\
\lim_{n \to \infty} \left( 2 - \frac{10(n+1)}{2n} + \frac{24(n+1)(2n+1)}{6n^2} - \frac{42}{n^2} \right) = \\
\lim_{n \to \infty} \left( 2 - 5 \cdot \frac{1}{n} + \frac{4 \cdot 1 \cdot 3 \cdot 1 \cdot 2}{n^2} - \frac{42}{n^2} \right) = \\
2 - 5 \cdot 1 + \frac{4 \cdot 1 \cdot 3 \cdot 1 \cdot 2}{1} - 0 = \\
5
\]

\[ \text{5 many ways to get this result} \]
4. (2 points) From section 5.2 we have the following property of definite integrals.

- If \( f(x) \) is continuous and \( m \leq f(x) \leq M \) for \( a \leq x \leq b \), then \( m(b-a) \leq \int_{a}^{b} f(x) \, dx \leq M(b-a) \)

Use this property to carefully explain why the following inequality holds.

\[
\frac{1}{20} \leq \frac{1}{\frac{1}{10}} \leq \frac{1}{10}
\]

\[
\frac{1}{10} \leq \frac{1}{\frac{1}{10}} \leq \frac{1}{10}
\]

\[
\frac{1}{20} \leq \frac{1}{\frac{1}{10}} \leq \frac{1}{10}
\]

\[
\frac{1}{20} \leq \frac{1}{\frac{1}{10}} \leq \frac{1}{10}
\]
5. (2 points) At time $t$ seconds, the velocity of an object is $v(t) = t^4 e^t$ m/s. The distance in meters traveled by this object from $t = 3$ to $t = 8$ can be written as a limit of Riemann sums in many different ways. I have shown how to do this for two of the six ways indicated below. Fill in the missing information for the remaining limits so that the only variables appearing are $n$ and $k$. Do not evaluate these limits.

(a) Using a limit of right Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( \frac{3+k\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+k\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$

(b) Using a limit of right Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( \frac{3+(k+1)\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+(k+1)\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$

(c) Using a limit of left Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( \frac{3+(k-1)\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+(k-1)\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$

(d) Using a limit of left Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( \frac{3+k\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+k\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$

(e) Using a limit of midpoint Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( \frac{3+(k-0.5)\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+(k-0.5)\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$

(f) Using a limit of midpoint Riemann sums,

$$ DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( \frac{3+(k+0.5)\cdot \frac{5}{n}}{n} \right)^4 e^{\left(3+(k+0.5)\cdot \frac{5}{n}\right)} \cdot \frac{5}{n} \right] $$