Math 220 (section AD?) Quiz 6 Fall 2015

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A spherical balloon is being inflated so that its diameter is increasing at a constant rate of 2 *cm/min*. How quickly is the volume of the balloon increasing when the diameter is 10 *cm* ?

D=diameter, r=radius, V=volume

Given: $\frac{dD}{dt} = 2$ cm/min

Want: $\frac{dV}{dt}\Big|_{D=10\text{ cm}}$

Note: $r = \frac{1}{2}D$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi\left(\frac{1}{2}D\right)^3$

$V = \frac{\pi}{6}D^3$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{6}D^3\right)$

$\frac{dV}{dt} = \frac{\pi}{2}D^2 \cdot \frac{dD}{dt}$ so at moment D=10cm,

$\frac{dV}{dt} = \frac{\pi}{2}(10)^2 \cdot (2)$

$\frac{dV}{dt} = 100\pi \frac{cm^3}{min}$

Volume is increasing by $100\pi \frac{cm^3}{min}$

2. (4 points) The position in meters of a particle at time $t \geq 0$ seconds is given by

$$s(t) = \frac{t^3}{3} - 2t^2 + 10t + 17$$

What is the particle's acceleration at the moment when the particle's velocity is 15 m/s ? Use correct units in your final answer.

(vel) $s'(t) = t^2 - 4t + 10$

set $s'(t) = 15$

$t^2 - 4t + 10 = 15$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5$ or $t = -1$ (but $t \geq 0 \Rightarrow t = 5$)

(acc) $s''(t) = 2t - 4$

$s''(5) = 6 \ m/s^2$

3. (3 points) A curve passes through the point $(1, e^5)$ and has the property that for each point on the curve, the slope of the curve is equal to twice the y-coordinate. What is the equation of the curve?

$\frac{dy}{dx} = 2y \Rightarrow y = Ce^{2x}$

At $(x,y) = (1, e^5)$, we have $e^5 = Ce^{2\cdot 1}$

$\Rightarrow C = \frac{e^5}{e^2} = e^3$

Thus $y = e^3 \cdot e^{2x}$

or

$y = e^{3+2x}$