1. (2 points) What is the slope of the curve \( y = 5e^x - 25 \tan(x) + 50 \sin(x) + 10x^2 - 100 \) at its y-intercept?

\[
\frac{dy}{dx} = 5e^x - 25 \sec^2(x) + 50 \cos(x) + 20x
\]

At the y-intercept, \( x = 0 \)

Thus,

\[
\text{slope} = \frac{dy}{dx} \bigg|_{x=0} = 5e^0 - 25 \sec^2(0) + 50 \cos(0) + 20 \cdot 0
\]

\[
= 5 - 25 + 50 = 30
\]

2. (2 points) There are an infinite number of points on the curve \( f(x) = -6 \cos x + 5x - 42 \) for which the line tangent to the curve is perpendicular to the line \( 2x + 16y = 5 \). Determine the x-value for at least one of these points.

\[
2x + 16y = 5 \implies y = -\frac{1}{8}x + \frac{5}{16}
\]

This line has slope \(-\frac{1}{8}\).

A line perpendicular to this line has slope \(-\frac{1}{-\frac{1}{8}} = 8\).

A line tangent to \( f(x) \) at \( x \) has slope

\[
f'(x) = 6 \sin(x) + 5.
\]

Thus \( 6 \sin(x) + 5 = 8 \implies \sin(x) = \frac{1}{2} \)

\[
x = \frac{\pi}{6}\] or more generally, \( x = \frac{\pi}{6} + 2k\pi \) for any integer \( k \).
3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dz}{dt}$, etc.), find derivatives for each of the following functions.

(a) $q = \left(\frac{x^{\sqrt{x}}}{\sqrt[3]{x}}\right)^{12} + \ln \left(\frac{\pi^3}{e^2 + 4}\right)$ (simplify your answer)

\[
\frac{dq}{dx} = 14x^{13}
\]

(b) $R = v^4 \csc(v)$

\[
\frac{dR}{dv} = \frac{d}{dv}(v^4 \csc(v)) + v^4 \frac{d}{dv}(\csc(v)) = 4v^3 \csc(v) + v^4 (-\csc(v) \cot(v)) = 4v^3 \csc(v) - v^4 \csc(v) \cot(v)
\]

(c) $w = \frac{5}{\sqrt{t} + 5e^t}$

\[
\frac{dw}{dt} = \frac{d}{dt}(5) \left(\sqrt{t} + 5e^t\right) - 5 \cdot \frac{d}{dt} \left(\sqrt{t} + 5e^t\right) = 0 \cdot (\sqrt{t} + 5e^t) - 5 \cdot \left(\frac{1}{2 \sqrt{t}} - \frac{5}{(\sqrt{t} + 5e^t)^2}\right) = \frac{-5}{2 \sqrt{t}} - 25e^t \frac{1}{(\sqrt{t} + 5e^t)^2}
\]