Math 220 (section AD?) Quiz 4 Fall 2015

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) What is the slope of the curve $y = 5e^x - 25\tan(x) + 50\sin(x) + 10x^2 - 100$ at its y-intercept?

$\frac{dy}{dx} = 5e^x - 25\sec^2(x) + 50\cos(x) + 20x$

At the y-intercept, $x = 0$

Thus,

$$\text{slope} = \left.\frac{dy}{dx}\right|_{x=0} = 5e^0 - 25\sec^2(0) + 50\cos(0) + 20\cdot 0$$
$$= 5 - 25 + 50$$
$$= 30$$

2. (2 points) There are an infinite number of points on the curve $f(x) = -6\cos x + 5x - 42$ for which the line tangent to the curve is perpendicular to the line $2x + 16y = 5$. Determine the x-value for at least one of these points.

$2x + 16y = 5 \Rightarrow y = -\frac{1}{8}x + \frac{5}{16}$

This line has slope $-1/8$.

A line perpendicular to this line has slope $\frac{-1}{-1/8} = 8$.

A line tangent to $f(x)$ at x has slope $f'(x) = 6\sin(x) + 5$.

Thus $6\sin(x) + 5 = 8 \Rightarrow \sin(x) = \frac{1}{2}$

$x = \frac{\pi}{6}$ or more generally, $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$ for any integer k

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $q = \left(\frac{x\sqrt{x}}{\sqrt[3]{x}}\right)^{12} + \ln\left(\frac{\pi^3}{e^2+4}\right)$ (simplify your answer)

$$q = \left(\frac{x^{3/2}}{x^{1/3}}\right)^{12} + \text{constant}$$

$$= \frac{(x^{3/2})^{12}}{(x^{1/3})^{12}} + \text{constant}$$

$$= \frac{x^{18}}{x^4} + \text{constant}$$

$$= x^{14} + \text{constant}$$

$$\frac{dq}{dx} = 14x^{13}$$

(b) $R = v^4 \csc(v)$

$$\frac{dR}{dv} = \frac{d}{dv}(v^4)\csc(v) + v^4\frac{d}{dv}(\csc(v))$$

$$= 4v^3\csc(v) + v^4(-\csc(v)\cot(v))$$

$$= 4v^3\csc(v) - v^4\csc(v)\cot(v)$$

(c) $w = \frac{5}{\sqrt{t}+5e^t}$

$$\frac{dw}{dt} = \frac{\frac{d}{dt}(5)\cdot(\sqrt{t}+5e^t) - 5\cdot\frac{d}{dt}(\sqrt{t}+5e^t)}{(\sqrt{t}+5e^t)^2}$$

$$= \frac{0\cdot(\sqrt{t}+5e^t) - 5\cdot(\frac{1}{2}t^{-1/2}+5e^t)}{(\sqrt{t}+5e^t)^2}$$

$$= \frac{-\frac{5}{2\sqrt{t}} - 25e^t}{(\sqrt{t}+5e^t)^2}$$