Math 220 Quiz 3 (take-home) Fall 2015

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Derek Jung
- ▷ **AD2**, TR 9:00-10:50, Claire Merriman
- ▷ **AD3**, TR 1:00-2:50, Itziar Ochoa de Alaiza Gracia
- ▷ **ADA**, TR 8:00-8:50, Dara Zirlin
- ▷ **ADB**, TR 9:00-9:50, Dara Zirlin
- ▷ **ADC**, TR 10:00-10:50, Xujun Liu
- ▷ **ADD**, TR 11:00-11:50, Christopher Linden
- ▷ **ADE**, TR 12:00-12:50, Christopher Linden
- ▷ **ADF**, TR 1:00-1:50, Alyssa Loving
- ▷ **ADG**, TR 2:00-2:50, Xianchang Meng
- ▷ **ADH**, TR 3:00-3:50, Xianchang Meng
- ▷ **ADI**, TR 4:00-4:50, Aaron Schneberger
- ▷ **ADJ**, TR 9:00-9:50, Elizabeth Field
- ▷ **ADK**, TR 10:00-10:50, Elizabeth Field
- ▷ **ADL**, TR 11:00-11:50, Emily Heath
- ▷ **ADM**, TR 12:00-12:50, Alyssa Loving
- ▷ **ADN**, TR 1:00-1:50, Aaron Schneberger
- ▷ **ADO**, TR 2:00-2:50, Tigran Hakobyan
- ▷ **ADP**, TR 3:00-3:50, Tigran Hakobyan
- ▷ **ADR**, TR 9:00-9:50, Xujun Liu
- ▷ **ADS**, TR 12:00-12:50, Emily Heath
- ▷ **ADT**, TR 2:00-2:50, Argen West
- ▷ **ADU**, TR 3:00-3:50, Argen West

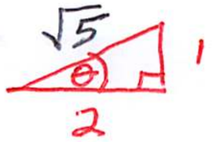
- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- **No technology is allowed. You are not allowed to use a calculator, search the Internet, use Wolfram Alpha, etc.**
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official discussion period on Thursday, September 17.**
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Thursday.**

1. (2 points) Evaluate $\sec(2\arctan(1/2))$. Write your answer as a simplified fraction.

Let $\theta = \arctan(1/2)$

Then $\tan\theta = \frac{1}{2}$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (actually $0 < \theta < \frac{\pi}{2}$ since $\tan\theta > 0$)

Pythagorean theorem $\rightarrow$




$$\sec(2\arctan(1/2)) = \sec(2\theta)$$
$$= \frac{1}{\cos(2\theta)}$$
$$= \frac{1}{\cos^2(\theta) - \sin^2(\theta)}$$
$$= \frac{1}{\left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2}$$
$$= \frac{1}{4/5 - 1/5}$$
$$= 5/3$$

2. (1 point) Which one of the following equations must hold in order for a function $z$ to be continuous at a number $d$ ?

(a) $\lim_{t\to 0} z(t) = d$

(b) $\lim_{t\to 0} z(t) = z(d)$

(c) $\lim_{t\to 0} z(t) = 0$

(d) $\lim_{t\to \infty} z(t) = d$

(e) $\lim_{t\to \infty} z(t) = z(d)$

(f) $\lim_{t\to \infty} z(t) = 0$

(g) $\lim_{t\to d} z(t) = d$

(h) $\lim_{t\to d} z(t) = z(d)$

(i) $\lim_{t\to d} z(t) = 0$

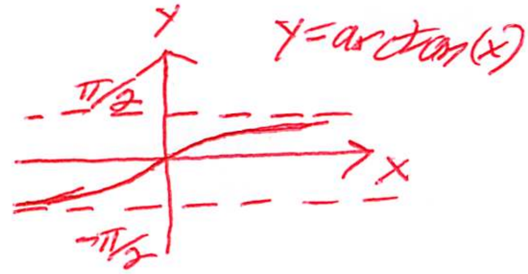
3. (2 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is $\infty$ or $-\infty$.

(a) $\lim\limits_{x\to-\infty} \dfrac{6\arctan(42x+5)}{e^x}$ $\quad \xrightarrow{} -3\pi$, $\xrightarrow{} 0^+$ $\quad = -\infty$

Justification

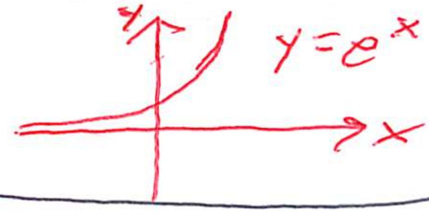
$\lim\limits_{x\to-\infty} (42x+5) = -\infty$

Thus, $\lim\limits_{x\to-\infty} \arctan(42x+5) = -\frac{\pi}{2}$




and $\lim\limits_{x\to-\infty} 6\arctan(42x+5) = 6\cdot -\frac{\pi}{2} = -3\pi$

also $\lim\limits_{x\to-\infty} e^x = 0$

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(b) $\lim\limits_{x\to 2} \dfrac{x-2}{3-\sqrt{2x+5}}$ $\quad \to 0$, $\to 0$ (indeterminate form)

$$\lim_{x\to 2} \frac{x-2}{3-\sqrt{2x+5}} = \lim_{x\to 2} \frac{x-2}{3-\sqrt{2x+5}} \cdot \frac{3+\sqrt{2x+5}}{3+\sqrt{2x+5}}$$

$$= \lim_{x\to 2} \frac{(x-2)(3+\sqrt{2x+5})}{3^2-(\sqrt{2x+5})^2}$$

$$= \lim_{x\to 2} \frac{(x-2)(3+\sqrt{2x+5})}{4-2x}$$

$$= \lim_{x\to 2} \frac{(x-2)(3+\sqrt{2x+5})}{-2(x-2)}$$

$$= \lim_{x\to 2} \frac{3+\sqrt{2x+5}}{-2} = \frac{3+\sqrt{2\cdot 2+5}}{-2} = -3$$

4. (3 points) Find all horizontal and vertical asymptotes on the graph of $f(x) = \dfrac{8e^{2x} - 27}{(2e^x - 3)^2}$

$$\lim_{x\to\infty} \frac{8e^{2x}-27 \to \infty}{(2e^x-3)^2 \to \infty} = \lim_{x\to\infty} \frac{8e^{2x}-27}{\left(e^x\left(2-\frac{3}{e^x}\right)\right)^2}$$

$$= \lim_{x\to\infty} \frac{e^{2x}\left(8-\frac{27}{e^{2x}}\right)}{e^{2x}\left(2-\frac{3}{e^x}\right)^2}$$

$$= \lim_{x\to\infty} \frac{8-27/e^{2x}}{(2-3/e^x)^2}$$

$$= \frac{8-0}{(2-0)^2} = 2$$

$$\lim_{x\to-\infty} \frac{8e^{2x}-27 \to -27}{(2e^x-3)^2 \to 9} = -3$$

(note: $\lim_{x\to-\infty}(e^{2x}) = \lim_{x\to-\infty}(e^x) = 0$


$\leftarrow$ shape of $y=e^{2x}$ and $y=e^x$ )

$\boxed{f(x) \text{ has horizontal asymptotes } y=2 \text{ and } y=-3}$

The denominator equals 0 when $2e^x-3=0 \Rightarrow x=\ln\left(\frac{3}{2}\right)$

Check if this gives a vertical asymptote.

$$\lim_{x\to\ln\left(\frac{3}{2}\right)} \frac{8e^{2x}-27 \to -9}{(2e^x-3)^2 \to 0^+} = -\infty$$

(note: $8e^{2\ln(3/2)}-27 = 8e^{\ln\left(\frac{3}{2}\right)^2}-27 = 8\cdot\frac{9}{4}-27 = -9$ )

$\boxed{f(x) \text{ has a vertical asymptote } x=\ln(3/2)}$