Math 220 (section AD?) **Quiz 2** **Fall 2015**

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) The following curves intersect. Find the x-value for each point of intersection.

$$y = 42 - \ln(5 + x)$$
$$y = 42 + \ln(5 - x)$$

$$42 + \ln(5-x) = 42 - \ln(5+x)$$
$$\ln(5-x) + \ln(5+x) = 0$$
$$\ln((5-x)(5+x)) = 0$$
$$e^{\ln((5-x)(5+x))} = e^0$$
$$(5-x)(5+x) = 1$$
$$25 - x^2 = 1$$
$$24 = x^2$$
$$x = \pm\sqrt{24} = \pm 2\sqrt{6}$$

Note: we also verify that both x-values are in the domains of the two given functions.

2. (3 points) Given that $w(x) = \dfrac{3+2x^3}{5-4x^3}$, find a formula for $w^{-1}(x)$.

$$y = \frac{3+2x^3}{5-4x^3}$$

$$x = \frac{3+2y^3}{5-4y^3}$$ (switch x & y)

$$x(5-4y^3) = 3+2y^3$$ (solve for y)

$$5x - 4xy^3 = 3+2y^3$$

$$5x-3 = 4xy^3 + 2y^3$$

$$5x-3 = y^3(4x+2)$$

$$\frac{5x-3}{4x+2} = y^3$$

$$y = \sqrt[3]{\frac{5x-3}{4x+2}}$$

$$w^{-1}(x) = \sqrt[3]{\frac{5x-3}{4x+2}}$$

3. (4 points) A bacterial culture starts with 300 bacteria and doubles in size every 5 hours.

(a) Find a formula for the number of bacteria as a function of the number of hours since its population was 300.

t	P
0	300
5	600
10	1200
15	2400
20	4800

$$P = C \cdot a^t$$
$$300 = C \cdot a^0 \Rightarrow C = 300$$
$$P = 300 \cdot a^t$$
$$600 = 300 \cdot a^5 \Rightarrow a^5 = 2 \Rightarrow a = 2^{1/5}$$
$$P = 300 \cdot (2^{1/5})^t$$
$$P = 300 \cdot 2^{t/5}$$

(b) At what time is the population equal to 2500 ?

From the table we see $15 < t < 20$.

To find the exact time, set $P = 2500$,

$$2500 = 300 \cdot 2^{t/5}$$
$$\frac{25}{3} = 2^{t/5}$$
$$\ln\left(\frac{25}{3}\right) = \ln\left(2^{t/5}\right)$$
$$\ln\left(\frac{25}{3}\right) = \frac{t}{5} \cdot \ln(2)$$
$$t = \frac{5 \ln\left(\frac{25}{3}\right)}{\ln(2)} \text{ hours}$$