Math 220 Quiz 12 (take-home) Fall 2015

Name Solutions

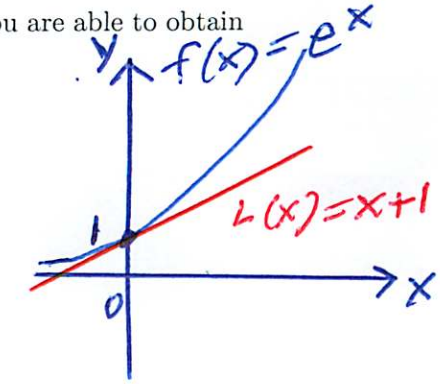
(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Derek Jung
- ▷ **AD2**, TR 9:00-10:50, Claire Merriman
- ▷ **AD3**, TR 1:00-2:50, Itziar Ochoa de Alaiza Gracia
- ▷ **ADA**, TR 8:00-8:50, Dara Zirlin
- ▷ **ADB**, TR 9:00-9:50, Dara Zirlin
- ▷ **ADC**, TR 10:00-10:50, Xujun Liu
- ▷ **ADD**, TR 11:00-11:50, Christopher Linden
- ▷ **ADE**, TR 12:00-12:50, Christopher Linden
- ▷ **ADF**, TR 1:00-1:50, Alyssa Loving
- ▷ **ADG**, TR 2:00-2:50, Xianchang Meng
- ▷ **ADH**, TR 3:00-3:50, Xianchang Meng
- ▷ **ADI**, TR 4:00-4:50, Aaron Schneberger
- ▷ **ADJ**, TR 9:00-9:50, Elizabeth Field
- ▷ **ADK**, TR 10:00-10:50, Elizabeth Field
- ▷ **ADL**, TR 11:00-11:50, Emily Heath
- ▷ **ADM**, TR 12:00-12:50, Alyssa Loving
- ▷ **ADN**, TR 1:00-1:50, Aaron Schneberger
- ▷ **ADO**, TR 2:00-2:50, Tigran Hakobyan
- ▷ **ADP**, TR 3:00-3:50, Tigran Hakobyan
- ▷ **ADR**, TR 9:00-9:50, Xujun Liu
- ▷ **ADS**, TR 12:00-12:50, Emily Heath
- ▷ **ADT**, TR 2:00-2:50, Argen West
- ▷ **ADU**, TR 3:00-3:50, Argen West

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic on problems 2 and 3.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official lecture period on Friday, November 20.**
- **Note to TAs and Tutors – you should not help students with these specific problems until I post solutions Friday evening.**

1. (3 points) A calculator gives an estimate of 0.8187307531 for the value of $\dfrac{e^2}{\sqrt[5]{e^{11}}}$

Using the techniques of linear approximation found in section 3.10, show that you are able to obtain a very similar estimate of 0.8 without the use of any technology.



$e^2/\sqrt[5]{e^{11}} = e^2/e^{11/5} = e^{2-11/5} = e^{-1/5}$

Tangent line to $f(x)=e^x$ at $x=0$

POINT: $(0, f(0)) = (0, e^0) = (0,1)$

SLOPE: $f'(0) = e^0 = 1$ (since $f'(x)=e^x$)

Tangent line: $L(x) = x+1$

$f(x) \approx L(x)$ for x near point of tangency

$e^x \approx x+1$ for x near 0

$e^{-1/5} \approx -\frac{1}{5}+1 = 0.8$ (from graph, we see this is an underestimate)

2. (3 points) Let $g(x) = \displaystyle\int_{-25}^{x^2} f(t)\,dt$. Use the techniques of linear approximation found in section 3.10 to approximate $g(4.8)$ given the following information about f.

- f is continuous on the interval $(-\infty, \infty)$ $\Rightarrow f$ is integrable on $(-\infty,\infty)$
- f is an odd function
- $f(25) = \dfrac{1}{8}$

Tangent line to $g(x)$ at $x=5$

Point: $g(5) = \int_{-25}^{25} f(t)\,dt = 0$ since f is odd

so point is $(5,0)$

slope: $g'(x) = f(x^2)\cdot 2x$ from F.T.C. part 1

$g'(5) = f(25)\cdot 10 = \frac{1}{8}\cdot 10 = \frac{5}{4}$

tangent line: $y - 0 = \frac{5}{4}(x-5) \Rightarrow L(x) = \frac{5}{4}(x-5)$

$g(x) \approx L(x)$ for x near point of tangency

$g(x) \approx \frac{5}{4}(x-5)$ for x near 5

$g(4.8) \approx \frac{5}{4}(4.8-5) \Rightarrow g(4.8) \approx -\frac{1}{4}$

3. (4 points) The function $g(x) = x^4 + 3x^2 - 5x$ has precisely one critical number. Determine the value of this critical number using Newton's Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$g'(x) = 4x^3 + 6x - 5$

This derivative exists for all x so the only critical numbers occur when $g'(x) = 0$. We need to solve $4x^3 + 6x - 5 = 0$

Let $f(x) = 4x^3 + 6x - 5$ (thus $f'(x) = 12x^2 + 6$) and apply Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{4x_n^3 + 6x_n - 5}{12x_n^2 + 6}$$

$x_1 = 1$ given as first estimate

$$x_2 = 1 - \frac{4(1)^3 + 6(1) - 5}{12(1)^2 + 6} = \frac{13}{18} \approx 0.7222222222$$

$$x_3 \approx 0.7222222222 - \frac{4(0.7222222222)^3 + 6(0.7222222222) - 5}{12(0.7222222222)^2 + 6}$$

$x_3 \approx 0.653686919 5$

$$x_4 \approx 0.6536869195 - \frac{4(0.6536869195)^3 + 6(0.6536869195) - 5}{12(0.6536869195)^2 + 6}$$

$x_4 \approx 0.6501443634$