Math 220 (section AD?) Quiz 11 Fall 2015

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = 6xe^{x^2}$ on the interval $[0, 5]$. Simplify your answer.

$$f_{ave} = \frac{1}{5-0}\int_0^5 6xe^{x^2}\,dx$$

$$= \frac{1}{5}\int_0^{25} 3e^u\,du$$

$$= \frac{3}{5}\left[e^u\right]_0^{25}$$

$$= \frac{3}{5}\left[e^{25} - e^0\right]$$

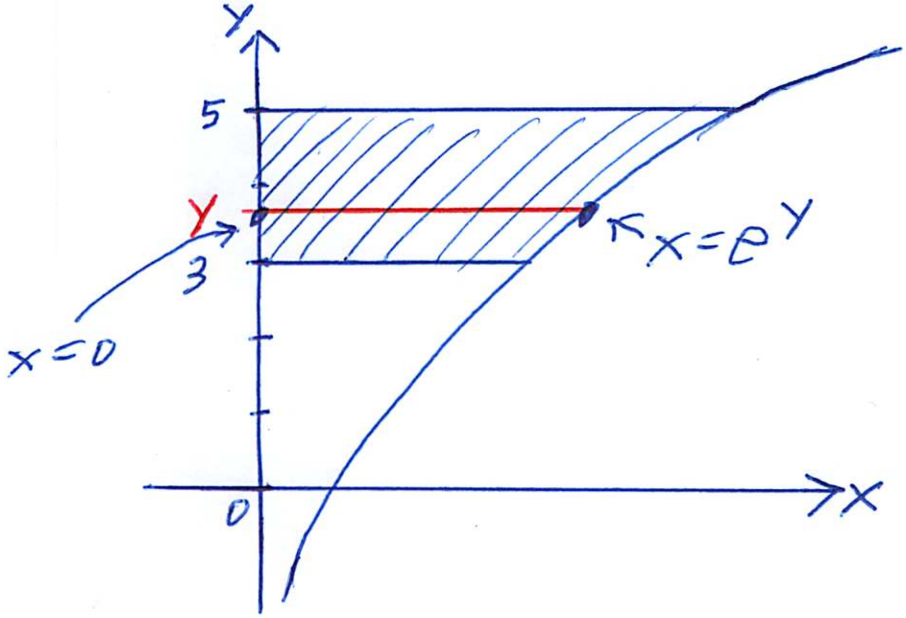
$$= \frac{3}{5}\left(e^{25} - 1\right)$$

$$\left(\begin{aligned} u &= x^2 \\ du &= 2x\,dx \\ 3\,du &= 6x\,dx \\ x = 0 &\Rightarrow u = 0^2 = 0 \\ x = 5 &\Rightarrow u = 5^2 = 25 \end{aligned}\right)$$

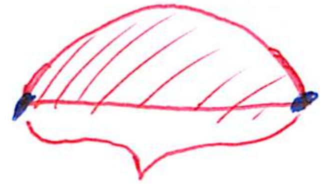
2. Let **R** be the finite region bounded by the graphs of $y = \ln x$, $y = 3$, $y = 5$ and $x = 0$. Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

   (a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the y-axis are semi-circles.

$y = \ln x \Rightarrow x = e^y$



<u>cross-section</u>



diameter $= e^y - 0$

radius $= \frac{1}{2}$ diameter

$= \frac{1}{2} e^y$

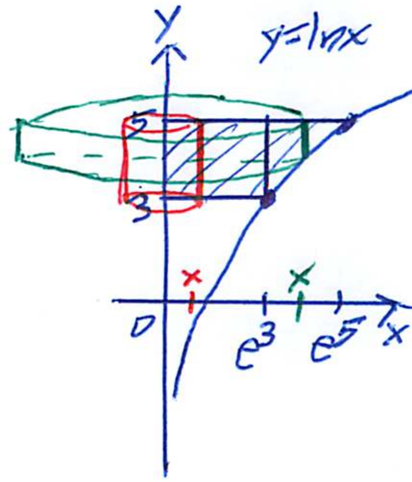
$$V = \int_3^5 (\text{cross-sectional area})\,dy$$

$$= \int_3^5 \frac{1}{2}\pi (\text{radius})^2\,dy$$

$$= \int_3^5 \frac{1}{2}\pi \left(\frac{1}{2}e^y\right)^2 dy$$

$$= \int_3^5 \frac{\pi}{8} e^{2y}\,dy$$

(b) The volume of the solid formed when **R** is revolved around the ~~line~~ y-axis. Determine this volume in the following two ways.



i. (2 points) Integrate with respect to x.

intersection points

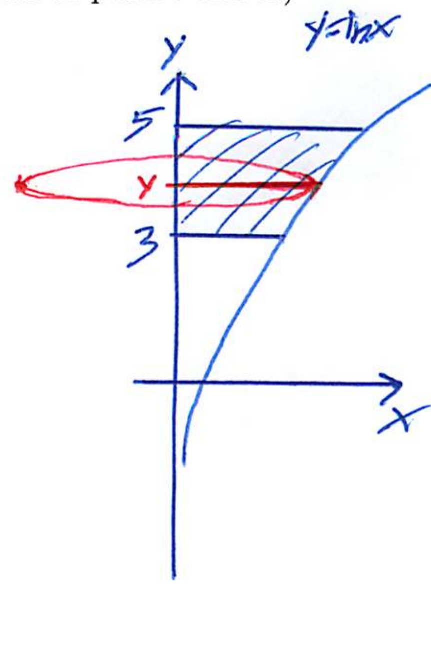
$y=3$ and $y=\ln x \Rightarrow \ln x = 3 \Rightarrow x = e^3$

$y=5$ and $y=\ln x \Rightarrow \ln x = 5 \Rightarrow x = e^5$

$V = \int_0^{e^5} (\text{surface area})\,dx$

$$V = \int_0^{e^3} 2\pi \underbrace{(x)}_{\text{rad}} \underbrace{(5-3)}_{\text{height}}\,dx + \int_{e^3}^{e^5} 2\pi \underbrace{(x)}_{\text{rad}} \underbrace{(5-\ln x)}_{\text{height}}\,dx$$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)



$V = \int_3^5 (\text{cross-sectional area})\,dy$

$V = \int_3^5 \pi \overbrace{(e^y - 0)}^{\text{rad.}}{}^2\,dy$

$$V = \int_3^5 \pi e^{2y}\,dy$$