Math 220 (section AD?) **Quiz 1** **Fall 2015**

Name: Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Given an acute angle θ for which $\tan(\theta) = 7$, evaluate the following quantities.

$\tan(\theta) = 7 = \frac{7}{1}$ $\left(\frac{opp}{adj}\right)$

$1^2 + 7^2 = (hyp)^2$

$hyp = \sqrt{50}$

(a) $\sec(\theta) = \frac{hyp}{adj} = \frac{\sqrt{50}}{1} = 5\sqrt{2}$

or use $\tan^2(\theta) + 1 = \sec^2(\theta)$

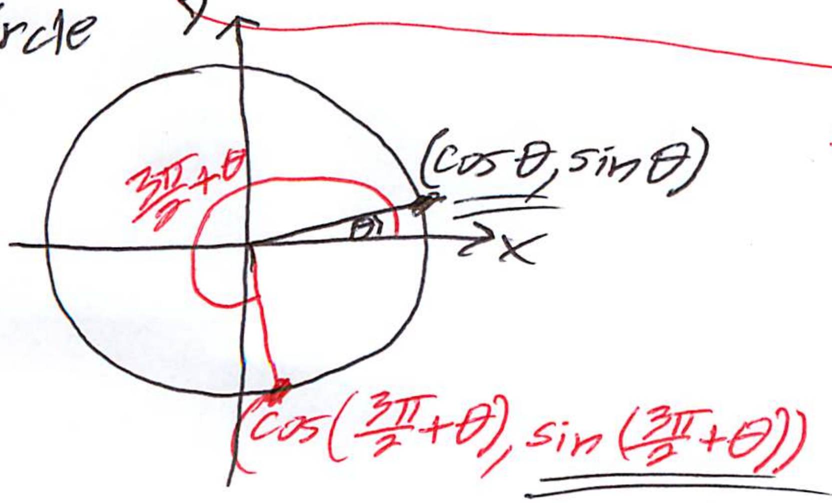
(b) $\cos(-\theta) = \cos(\theta)$ since $\cos(\theta)$ is an even function

$= \frac{adj}{hyp}$

$= \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$

(c) $\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta = -\frac{1}{\sqrt{50}} = \frac{-1}{5\sqrt{2}} = -\frac{\sqrt{2}}{10}$

unit circle



or use

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin\left(\frac{3\pi}{2}+\theta\right) = \sin\left(\frac{3\pi}{2}\right)\cos(\theta) + \cos\left(\frac{3\pi}{2}\right)\sin(\theta)$

$= -1 \cdot \cos(\theta) + 0 \cdot \sin(\theta)$

$= -\cos(\theta)$

$= -\frac{1}{\sqrt{50}} = \frac{-1}{5\sqrt{2}} = \frac{-\sqrt{2}}{10}$

2. (4 points) Determine the domain of the given function.

$$f(x) = \frac{3\sin(x-5) + 4\cos(x-1)}{10 - \sqrt{100-(x-2)^2}}$$

no restrictions on domain from this numerator

$100-(x-2)^2 \geq 0 \Rightarrow (x-2)^2 \leq 100$

$\Rightarrow \sqrt{(x-2)^2} \leq \sqrt{100}$

$\Rightarrow |x-2| \leq 10$

$\Rightarrow -10 \leq x-2 \leq 10$

$\Rightarrow -8 \leq x \leq 12$

The denominator equals 0 when

$10 - \sqrt{100-(x-2)^2} = 0$

$10 = \sqrt{100-(x-2)^2}$

$100 = 100-(x-2)^2$

$(x-2)^2 = 0$

$x = 2$

DOMAIN OF $f(x)$ IS

$[-8, 2) \cup (2, 12]$

3. (3 points) Suppose that $f(x)$ is an even function, $g(x)$ is an odd function, and the function $h(x)$ given below has a non-empty domain. Prove that $h(x)$ is an odd function.

$$h(x) = \frac{(f \circ g)(x)}{f(x)g(x)}$$

$$h(-x) = \frac{(f \circ g)(-x)}{f(-x)g(-x)}$$

$$= \frac{f(g(-x))}{f(x) \cdot (-g(x))} \leftarrow \text{since } f \text{ is even and } g \text{ is odd}$$

$$= \frac{f(-g(x))}{-f(x) \cdot g(x)} \leftarrow \text{since } g \text{ is odd}$$

$$= \frac{f(g(x))}{-f(x) \cdot g(x)} \leftarrow \text{since } f \text{ is even}$$

$$= -\frac{(f \circ g)(x)}{f(x) \cdot g(x)}$$

$$= -h(x)$$

Thus, $h(x)$ is an odd function