Math 220 – Test 3 Information

The test will be given during your lecture period on Wednesday (December 3, 2014). No books, notes, scratch paper, calculators or other electronic devices are allowed. Bring a Student ID.

It may be helpful to look at:

- [http://www.math.illinois.edu/~murphyrf/teaching/M220-F2014/](http://www.math.illinois.edu/~murphyrf/teaching/M220-F2014/) – Volumes worksheet, quizzes 8, 9, 10, 11 and 12, and Daily Assignments for a summary of each lecture
- [https://compass2g.illinois.edu/](https://compass2g.illinois.edu/) – Homework solutions
- [http://www.math.illinois.edu/~murphyrf/teaching/M220/](http://www.math.illinois.edu/~murphyrf/teaching/M220/) – Tests and quizzes in my previous courses

- **Section 3.10 (Linear Approximation and Differentials)**
  - Be able to use a tangent line (or differentials) in order to approximate the value of a function near the point of tangency.

- **Section 4.2 (The Mean Value Theorem)**
  - Be able to precisely state The Mean Value Theorem and Rolle’s Theorem.
  - Be able to decide when functions satisfy the conditions of these theorems. If a function does satisfy the conditions, then be able to find the value of $c$ guaranteed by the theorems.
  - Be able to use The Mean Value Theorem, Rolle’s Theorem, or earlier important theorems such as The Intermediate Value Theorem to prove some other fact. In the homework these often involved roots, solutions, $x$-intercepts, or intersection points.

- **Section 4.8 (Newton’s Method)**
  - Understand the graphical basis for Newton’s Method (that is, use the point where the tangent line crosses the $x$-axis as your next estimate for a root of a function).
  - Be able to apply Newton’s Method to approximate roots, solutions, $x$-intercepts, or intersection points.

- **Section 4.9 (Antiderivatives)**
  - Know antiderivative formulas for $0, k$ (a constant), $\sin x$, $\cos x$, $\sec^2 x$, $\csc^2 x$, $\sec x \tan x$, $\csc x \cot x$, $e^x$, $\frac{1}{1 + x^2}$, $\frac{1}{\sqrt{1 - x^2}}$, $x^n \ (n \neq -1)$, $x^{-1} = \frac{1}{x}$.
  - Be able to find general antiderivatives for functions which are sums or differences of constants multiplied by the above formulas (you may need to simplify first).
  - Be able to solve a differential equation where values for the function or its first or second derivative are given.
  - Be able to apply these rules to problems involving acceleration, velocity, or position.
  - You should know that the acceleration due to gravity is approximately $-32 \ ft/s^2$ or $-9.8 \ m/s^2$ near the Earth’s surface.
• Section 5.1 (Areas and Distances)
  – Use Riemann sums (left, right, or midpoint) to estimate area or total change in a quantity and state if your estimate is known to be an underestimate or overestimate. These sums will involve at most 8 subintervals.
  – Use limits of Riemann sums to find the exact area or total change in a quantity. Being able to do this with right Riemann sums will be sufficient for this test.
  – Understand sigma notation for sums and know the following sums.
    \[ \sum_{k=1}^{n} C = C \cdot n \text{ (C is a constant)} \]
    \[ \sum_{k=1}^{n} k = \frac{n(n + 1)}{2} \]
    \[ \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6} \]
    \[ \sum_{k=1}^{n} k^3 = \left( \frac{n(n + 1)}{2} \right)^2 \]

• Section 5.2 (The Definite Integral)
  – Understand the definition of a definite integral as \( \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x \). Be able to more explicitly write out the appropriate limit for specific functions on given intervals. You may have to evaluate one such limit.
  – Know the relationship between a definite integral and area. This should be understood regardless of whether or not the graph of the function being integrated is above or below the \( x \)-axis.
  – Know the following properties of the definite integral.
    \[ \int_{b}^{a} f(x) \, dx = - \int_{a}^{b} f(x) \, dx \]
    \[ \int_{a}^{a} f(x) \, dx = 0 \]
    \[ \int_{a}^{b} c \, dx = c(b - a) \text{ where } c \text{ is any constant} \]
    \[ \int_{a}^{b} [f(x) + g(x)] \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \]
    \[ \int_{a}^{b} cf(x) \, dx = c \int_{a}^{b} f(x) \, dx \text{ where } c \text{ is any constant} \]
    \[ \int_{a}^{b} [f(x) - g(x)] \, dx = \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx \]
    \[ \int_{a}^{c} f(x) \, dx + \int_{c}^{b} f(x) \, dx = \int_{a}^{b} f(x) \, dx \]
    \[ \text{If } f(x) \geq 0 \text{ for } a \leq x \leq b, \text{ then } \int_{a}^{b} f(x) \, dx \geq 0 \]
    \[ \text{If } f(x) \geq g(x) \text{ for } a \leq x \leq b, \text{ then } \int_{a}^{b} f(x) \, dx \geq \int_{a}^{b} g(x) \, dx \]
    \[ \text{If } m \leq f(x) \leq M \text{ for } a \leq x \leq b, \text{ then } m(b - a) \leq \int_{a}^{b} f(x) \, dx \leq M(b - a) \]
• Section 5.3 (The Fundamental Theorem of Calculus)
  – Be able to precisely state Part 1 and Part 2 of the The Fundamental Theorem of Calculus.
  – When the conditions of the theorem hold, be able to use Part 1 to find the derivative of functions which are defined in terms of integrals.
  – When the conditions of the theorem hold, be able to use Part 2 to evaluate definite integrals.

• Section 5.4 (Indefinite Integrals and the Net Change Theorem)
  – Know indefinite integral formulas for 0, k (a constant), sin x, cos x, sec^2 x, csc^2 x, sec x tan x, csc x cot x, e^x, \frac{1}{1 + x^2}, \frac{1}{\sqrt{1 - x^2}}, x^n (n \neq -1), x^{-1} = \frac{1}{x}.
  – Know that the definite integral of a rate of change gives the total change. Be able to use this Net Change Theorem for applied problems involving rates of change such as velocity, acceleration, growth rates, etc.

• Section 5.5 (The Substitution Rule)
  – Be able to solve a wide variety of definite or indefinite integrals using substitution.
  – Be able to more quickly evaluate definite integrals on the interval \([-a, a]\) given that the integrand is continuous and either even or odd on that interval.

• Section 6.1 (Areas between Curves)
  – Be able to find areas between curves. This may require breaking the area up into the sum of two or more definite integrals.
  – Be able to integrate with respect to x or with respect to y to determine these areas.

• Section 6.2 (Volumes)
  – Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  – Be able to find volumes for solids formed by building upon some base and having cross-sections which are rectangles, squares, triangles, semi-circles, etc.
  – Be able to integrate with respect to x or with respect to y to determine these volumes.

• Section 6.3 (Volumes by Cylindrical Shells)
  – Be able to find volumes for solids formed by revolving a region around any vertical or horizontal line.
  – Be able to integrate with respect to x or with respect to y to determine these volumes.
• Section 6.5 (Average Value of a Function)
  – Be able to find the average value of a function.
  – Know the graphical interpretation of the average value of a function.
  – Know The Mean Value Theorem for Integrals.

• Section 7.2 (Trigonometric Integrals)
  – Be able to use substitution to solve definite or indefinite integrals involving trigonometric functions.
  – Be able to use basic trigonometric definitions and identities to help evaluate these integrals. In particular be able to use

\[
\begin{align*}
\ast \tan x &= \frac{\sin x}{\cos x} \\
\ast \cot x &= \frac{\cos x}{\sin x} \\
\ast \sec x &= \frac{1}{\cos x} \\
\ast \csc x &= \frac{1}{\sin x} \\
\ast \sin^2 x + \cos^2 x &= 1 \\
\ast \tan^2 x + 1 &= \sec^2 x \\
\ast 1 + \cot^2 x &= \csc^2 x \\
\ast \sin (2x) &= 2 \sin x \cos x \\
\ast \cos (2x) &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \\
\ast \cos^2 x &= \frac{1}{2} + \frac{1}{2} \cos (2x) \\
\ast \sin^2 x &= \frac{1}{2} - \frac{1}{2} \cos (2x)
\end{align*}
\]