Math 220 (section AD?) Quiz 9 Fall 2014

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following indefinite integrals.

(a) $\int (3x - 10\sqrt{x})^2 \, dx = \int \left((3x)^2 - 2(3x)(10\sqrt{x}) + (10\sqrt{x})^2\right) dx$

$= \int (9x^2 - 60x^{3/2} + 100x) dx$

$= 9 \cdot \frac{1}{3}x^3 - 60 \cdot \frac{1}{5/2} x^{5/2} + 100 \cdot \frac{1}{2} x^2 + C$

$= 3x^3 - 24x^{5/2} + 50x^2 + C$

(b) $\int \frac{x^2+10}{x^2+1} \, dx = \int \frac{x^2+1+9}{x^2+1} dx = \int \left(\frac{x^2+1}{x^2+1} + \frac{9}{x^2+1}\right) dx$

$= \int \left(1 + 9 \cdot \frac{1}{x^2+1}\right) dx$

$= x + 9\arctan(x) + C$

note: you can also use polynomial long division

$$\begin{array}{r} 1 \\ x^2+1 \overline{\smash{)}\, x^2+10} \\ \underline{x^2+1} \\ 9 \end{array}$$

$\frac{x^2+10}{x^2+1} = 1 + \frac{9}{x^2+1}$

2. (2 points) Evaluate the following definite integral. Simplify your answer.

$\int_{\pi/6}^{\pi/3} \frac{\cos^3 x \sin^2 x + \cos^5 x}{1 - \sin^2 x} \, dx = \int_{\pi/6}^{\pi/3} \frac{\cos^3 x(\sin^2 x + \cos^2 x)}{\cos^2 x} dx$

$= \int_{\pi/6}^{\pi/3} \cos x \, dx$ since $\sin^2 x + \cos^2 x = 1$

$= [\sin x]_{\pi/6}^{\pi/3}$

$= \sin(\pi/3) - \sin(\pi/6)$

$= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

note: you can get $p'(x)$ more quickly as done in lecture

3. (2 points) Suppose $p(x) = \int_8^{x^3-75x} \frac{1}{t^4+100}\,dt$. Determine each interval upon which the graph of $p(x)$ is decreasing.

Let $u = x^3 - 75x$. Then $\frac{du}{dx} = 3x^2 - 75$

$p = \int_8^u \frac{1}{t^4+100}\,dt \Rightarrow \frac{dp}{du} = \frac{1}{u^4+100}$ by Fundamental Theorem of calculus (Part 1)

$p'(x) = \frac{dp}{dx} = \frac{dp}{du} \cdot \frac{du}{dx}$ by the chain rule

$= \frac{1}{u^4+100} \cdot (3x^2-75) = \frac{3x^2-75}{(x^3-75x)^4+100}$

$= \frac{3(x+5)(x-5)}{(x^3-75x)^4+100}$

values of $p'(x)$

+++ 0 --- 0 +++ → x
-5 5

p(x) is decreasing on the interval $[-5, 5]$

4. (2 points) An oil storage tank ruptures and oil leaks from the tank at the rate of $r(t) = 3\sqrt{t}$ liters per minute where t is measured in minutes since the tank was ruptured. How much oil leaks out of the tank during the first 25 minutes?

Let $a(t)$ = amount of oil outside the tank.

Then $a'(t) = r(t) = 3\sqrt{t}$ liters/min.

Note that $\int_0^{25} a'(t)\,dt = a(25) - a(0)$

is the amount of oil which leaked out of tank.

Thus amount of oil leaked $= \int_0^{25} 3\sqrt{t}\,dt = \int_0^{25} 3t^{1/2}\,dt$

$= \left[2t^{3/2}\right]_0^{25}$

$= 2(25)^{3/2} - 2(0)^{3/2} = 250$ liters

Note: we are using the net change theorem (ie, fundamental theorem of calculus - part 2)