Math 220 (section AD?) Quiz 6 Fall 2014

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) A man is standing on a bridge over a river. He reaches over the railing and throws a stone vertically upward. Until it lands in the river, the stone's height in feet above the river is $h = -16t^2 + 8t + 24$ where t is measured in seconds since the stone was thrown.

   (a) What is the maximum height reached by the stone?

(height) $h = -16t^2 + 8t + 24$

(velocity) $h' = -32t + 8$

values of h': $+++0---$ on the t-axis, with 0 at $t = \frac{1}{4}$

at max height, $h' = 0$

$0 = -32t + 8 \Rightarrow t = \frac{1}{4}$ sec.

$h(\frac{1}{4}) = -16(\frac{1}{4})^2 + 8(\frac{1}{4}) + 24$

$= 25$ feet

The stone reaches a max height of 25 ft $\frac{1}{4}$ sec after it was thrown.

   (b) What is the velocity of the stone as it strikes the river?

set $h = 0$

$0 = -16t^2 + 8t + 24$

$0 = -8(2t^2 - t - 3)$

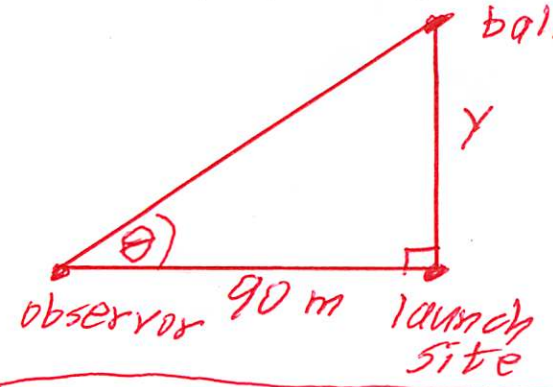
$0 = -8(2t - 3)(t + 1)$

$t = \frac{3}{2}$ or $t = -1$

only $t > 0$ makes sense so $t = \frac{3}{2}$ sec

$h'(\frac{3}{2}) = -32(\frac{3}{2}) + 8 = -40$ ft/sec

2. (3 points) There is a launch site of a hot-air balloon on the ground 90 meters away from an observor. The balloon rises vertically at a constant rate of 3 meters per second. How quickly is the angle of elevation of the balloon increasing 40 seconds after the launch?



Given: $\frac{dy}{dt} = 3$ m/s

Find: $\left.\frac{d\theta}{dt}\right|_{t=40\,s}$

$\tan\theta = \frac{y}{90}$

$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{y}{90}\right)$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{90}\frac{dy}{dt}$

at $t = 40$ s, we see that

$y = (3\ \text{m/s})(40\ \text{s}) = 120$ m

$\Rightarrow \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{90^2+120^2}}{90} = \frac{150}{90} = \frac{5}{3}$

$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{1}{90}\cdot 3 \Rightarrow \frac{d\theta}{dt} = \frac{3}{250}$ rad/s ★

3. (3 points) Suppose that A represents the number of grams of a radioactive substance at time t seconds. Given that $\frac{dA}{dt} = -0.25A$, how long does it take 12 grams of this substance to be reduced to 7 grams?

$\frac{dA}{dt} = -0.25A \Rightarrow A = Ce^{-0.25t}$

$A(0) = 12 \Rightarrow 12 = Ce^{-0.25(0)}$

$\Rightarrow C = 12$

$A = 12e^{-0.25t}$

$7 = 12e^{-0.25t}$

$\frac{7}{12} = e^{-0.25t}$

$\ln(7/12) = \ln(e^{-0.25t}) = -0.25t$

$t = \frac{\ln(7/12)}{-0.25}$ seconds