Math 220 (section AD?) **Quiz 5** **Fall 2014**

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Given $g(t) = e^{t^3}$, find its second derivative $g''(t)$.

$$g'(t) = e^{t^3} \cdot \frac{d}{dt}(t^3) = e^{t^3} \cdot 3t^2$$

$$g''(t) = \frac{d}{dt}(e^{t^3}) \cdot (3t^2) + (e^{t^3}) \cdot \frac{d}{dt}(3t^2)$$

$$g''(t) = (e^{t^3} \cdot 3t^2) \cdot (3t^2) + (e^{t^3}) \cdot (6t)$$

$$g''(t) = 9t^4 e^{t^3} + 6t e^{t^3}$$

$$g''(t) = 3t e^{t^3}(3t^3 + 2)$$

both answers acceptable

2. (2 points) Compute $f'(r)$ given that $f(r) = \arctan\left(\sqrt{r^2+5}\right)$.

$$f'(r) = \frac{1}{(\sqrt{r^2+5})^2 + 1} \cdot \frac{d}{dr}(\sqrt{r^2+5})$$

$$f'(r) = \frac{1}{r^2+6} \cdot \frac{1}{2}(r^2+5)^{-1/2} \cdot \frac{d}{dr}(r^2+5)$$

$$f'(r) = \frac{1}{r^2+6} \cdot \frac{1}{2\sqrt{r^2+5}} \cdot 2r$$

$$f'(r) = \frac{r}{(r^2+6)\sqrt{r^2+5}}$$

3. (2 points) Compute $\frac{dy}{dx}$ given that $y = (\sin x)^{5x^2}$.

See last page for alternate approach

$\ln(y) = \ln((\sin x)^{5x^2})$

$\ln(y) = 5x^2 \cdot \ln(\sin x)$

$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(5x^2 \cdot \ln(\sin x))$

$\frac{1}{y} \cdot \frac{dy}{dx} = 10x \cdot \ln(\sin x) + 5x^2 \cdot \frac{1}{\sin x} \cdot \cos x$

$\frac{dy}{dx} = y\left[10x\ln(\sin x) + 5x^2 \cot x\right]$

$\frac{dy}{dx} = (\sin x)^{5x^2}\left[10x\ln(\sin x) + 5x^2 \cot x\right]$

4. (3 points) Find the equation of the line tangent to the curve $5y^4 - x^3 = x^2y - 7$ at the point $(x, y) = (2, 1)$.

$\frac{d}{dx}(5y^4 - x^3) = \frac{d}{dx}(x^2y - 7)$

$20y^3\frac{dy}{dx} - 3x^2 = \frac{d}{dx}(x^2)(y) + (x^2)\frac{d}{dx}(y) - 0$

$20y^3\frac{dy}{dx} - 3x^2 = 2xy + x^2\frac{dy}{dx}$

$20y^3\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy + 3x^2$

$\frac{dy}{dx}(20y^3 - x^2) = 2xy + 3x^2$

$\frac{dy}{dx} = \frac{2xy + 3x^2}{20y^3 - x^2}$

$\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{2\cdot 2\cdot 1 + 3\cdot 2^2}{20\cdot 1^3 - 2^2} = \frac{16}{16} = 1$ (slope)

POINT: $(2,1)$

SLOPE: 1

TANGENT LINE:

$y - 1 = 1\cdot(x-2)$

$y = x - 1$

#3

alternate approach

$$y = (\sin x)^{5x^2}$$

$$y = e^{\ln\left((\sin x)^{5x^2}\right)}$$

$$y = e^{5x^2 \cdot \ln(\sin x)}$$

$$\frac{dy}{dx} = e^{5x^2 \cdot \ln(\sin x)} \cdot \frac{d}{dx}\left(5x^2 \cdot \ln(\sin x)\right)$$

$$\frac{dy}{dx} = e^{5x^2 \cdot \ln(\sin x)} \cdot \left(10x \cdot \ln(\sin x) + 5x^2 \cdot \frac{1}{\sin x} \cdot \cos x\right)$$

$$\frac{dy}{dx} = e^{\ln\left((\sin x)^{5x^2}\right)} \cdot \left(10x \cdot \ln(\sin x) + 5x^2 \cot x\right)$$

$$\frac{dy}{dx} = (\sin x)^{5x^2}\left(10x\ln(\sin x) + 5x^2 \cot x\right)$$

#3

Alternate approach uses that

$$u = e^{\ln(u)}$$