Math 220 (section AD?) Quiz 4 Fall 2014

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) What is the slope of the curve $y = 5 - 2e^x$ at its x-intercept? Simplify your answer.

To find x-intercept, set $y=0$

$0 = 5 - 2e^x$

$2e^x = 5$

$e^x = \frac{5}{2}$

$x = \ln(\frac{5}{2})$

$y' = -2e^x$

slope at $x = \ln(\frac{5}{2})$

is $y'(\ln(\frac{5}{2})) = -2e^{\ln(\frac{5}{2})} = -2 \cdot \frac{5}{2} = -5$

2. (2 points) Find the equation of the line which is tangent to the curve $f(x) = \sqrt{x}$ and parallel to the line $x - 10y = 40$.

$x - 10y = 40 \Rightarrow y = \frac{1}{10}x - 4$ which has slope $\frac{1}{10}$.

Parallel lines have equal slopes, so we must find a tangent line with slope $\frac{1}{10}$.

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{1}{10} = \frac{1}{2\sqrt{x}} \Rightarrow x = 25$

Point: $(25, f(25)) = (25, 5)$

SLOPE: $1/10$

TANGENT LINE: $y - 5 = \frac{1}{10}(x - 25) \Rightarrow y = \frac{1}{10}x + \frac{5}{2}$

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $\alpha = 5e^t + 2\csc(t) - 6\tan(t) + \underbrace{\sin(2\arctan(8/15))}_{\text{constant with derivative } 0}$

$$\frac{d\alpha}{dt} = 5e^t - 2\csc(t)\cot(t) - 6\sec^2(t)$$

(b) $w = \left(\frac{x\sqrt{x}}{\sqrt[3]{x^4}}\right)^{24}$ (simplify your answer)

$$w = \left(\frac{x^{3/2}}{x^{4/3}}\right)^{24}$$

$$w = \frac{(x^{3/2})^{24}}{(x^{4/3})^{24}}$$

$$w = \frac{x^{36}}{x^{32}}$$

$$w = x^4$$

$$\frac{dw}{dx} = 4x^3$$

(c) $H = \frac{2 + 5\cos(r)}{r^8 + 42}$

$$\frac{dH}{dr} = \frac{\frac{d}{dr}(2+5\cos(r))\cdot(r^8+42) - (2+5\cos(r))\cdot\frac{d}{dr}(r^8+42)}{(r^8+42)^2}$$

$$\frac{dH}{dr} = \frac{-5\sin(r)\cdot(r^8+42) - (2+5\cos(r))\cdot 8r^7}{(r^8+42)^2}$$