Math 220 (section AD?) Quiz 3 Fall 2014

Name SOLUTIONS

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Determine the equation for one of the horizontal asymptotes on the graph of

$$f(x) = \frac{1}{\sqrt{36x^2+3x}-6x}$$

$$\lim_{x\to\infty} \frac{1}{\sqrt{36x^2+3x}-6x} = \lim_{x\to\infty}\left(\frac{1}{\sqrt{36x^2+3x}-6x}\cdot\frac{\sqrt{36x^2+3x}+6x}{\sqrt{36x^2+3x}+6x}\right)$$

$$= \lim_{x\to\infty}\frac{\sqrt{36x^2+3x}+6x}{(\sqrt{36x^2+3x})^2-(6x)^2} = \lim_{x\to\infty}\frac{\sqrt{36x^2+3x}+6x}{3x}$$

$$= \lim_{x\to\infty}\frac{\sqrt{36x^2+3x}+6x}{3x}\cdot\frac{1/x}{1/x} = \lim_{x\to\infty}\frac{\frac{\sqrt{36x^2+3x}}{x}+6}{3}$$

$$= \lim_{x\to\infty}\frac{\frac{\sqrt{36x^2+3x}}{\sqrt{x^2}}+6}{3} = \lim_{x\to\infty}\frac{\sqrt{36+\frac{3}{x}}+6}{3} = \frac{\sqrt{36}+6}{3} = 4$$

horizontal asymptote at $y=4$

as $x\to-\infty$, we mimic the beginning lines above getting

$$\lim_{x\to-\infty}\frac{1}{\sqrt{36x^2+3x}-6x} = \cdots = \lim_{x\to-\infty}\frac{\frac{\sqrt{36x^2+3x}}{x}+6}{3}$$

$$= \lim_{x\to-\infty}\frac{\frac{\sqrt{36x^2+3x}}{-\sqrt{x^2}}+6}{3} = \lim_{x\to-\infty}\frac{-\sqrt{36+\frac{3}{x}}+6}{3} = 0$$

hor. asymp. at $y=0$

2. (1 point) Which one of the following equations must hold in order for a function g to be continuous at a number k ?

(a) $\lim_{x\to\infty} g(x) = g(k)$	(b) $\lim_{x\to\infty} g(x) = 0$	(c) $\lim_{x\to\infty} g(x) = k$
(d) $\lim_{x\to 0} g(x) = g(k)$	(e) $\lim_{x\to 0} g(x) = 0$	(f) $\lim_{x\to 0} g(x) = k$
(g) $\lim_{x\to k} g(x) = g(k)$	(h) $\lim_{x\to k} g(x) = 0$	(i) $\lim_{x\to k} g(x) = k$

3. (2 points) Evaluate $\csc(\arctan(0.5))$.

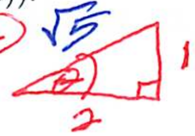
① Let $\theta = \arctan(0.5)$

Then $\tan\theta = 0.5$

with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(actually $0 < \theta < \frac{\pi}{2}$ since $\tan\theta > 0$)

②



$\tan\theta = 0.5 = \frac{1}{2}$ $\left(\frac{opp}{adj}\right)$

From Pyth. Thm,

$1^2 + 2^2 = hyp^2$,

$\Rightarrow hyp = \sqrt{5}$

③ $\csc(\arctan(0.5))$

$= \csc(\theta)$

$= \frac{\sqrt{5}}{1}$ $\left(\frac{hyp}{opp}\right)$

$= \sqrt{5}$

4. (2 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\displaystyle\lim_{x \to 2^+} \frac{x^2 - 7x + 10}{x^2 - 4x + 4}$ $\begin{matrix}\to 0\\ \to 0\end{matrix}$ ($\frac{0}{0}$ is an indeterminate form)

$$\lim_{x \to 2^+} \frac{x^2 - 7x + 10}{x^2 - 4x + 4} = \lim_{x \to 2^+} \frac{(x-2)(x-5)}{(x-2)^2}$$

$$= \lim_{x \to 2^+} \frac{x-5}{x-2} \begin{matrix}\to -3\\ \to 0^+\end{matrix}$$

$$= -\infty$$

(b) $\displaystyle\lim_{x \to \infty} \frac{(2e^x + 3)^2}{5e^{2x} + 1}$ $\begin{matrix}\to \infty\\ \to \infty\end{matrix}$ ($\frac{\infty}{\infty}$ is an indeterminate form)

$$\lim_{x \to \infty} \frac{(2e^x + 3)^2}{5e^{2x} + 1} = \lim_{x \to \infty} \frac{(2e^x)^2 + 2(2e^x)(3) + (3)^2}{5e^{2x} + 1}$$

$$= \lim_{x \to \infty} \frac{4e^{2x} + 12e^x + 9}{5e^{2x} + 1} \cdot \frac{1/e^{2x}}{1/e^{2x}}$$

$$= \lim_{x \to \infty} \frac{4 + \frac{12}{e^x} + \frac{9}{e^{2x}}}{5 + \frac{1}{e^{2x}}} \begin{matrix}\to 4\\ \to 5\end{matrix}$$

$$= 4/5$$