

Name

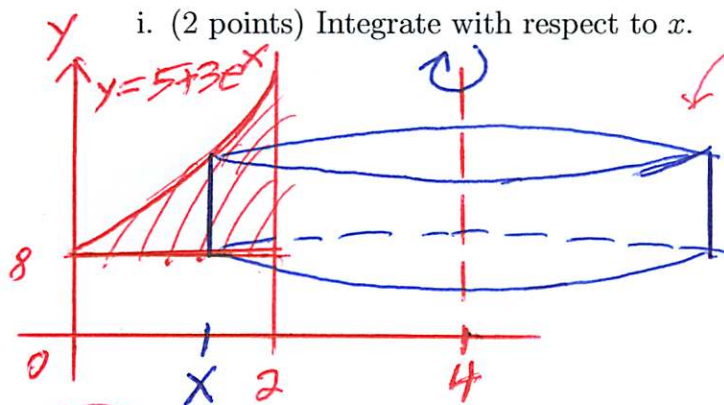
Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. Let R be the finite region bounded by the graphs of $y = 5 + 3e^x$, $y = 8$ and $x = 2$. Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

(a) The volume of the solid formed when R is revolved around the vertical line $x = 4$. Determine this volume in the following two ways.

i. (2 points) Integrate with respect to x .



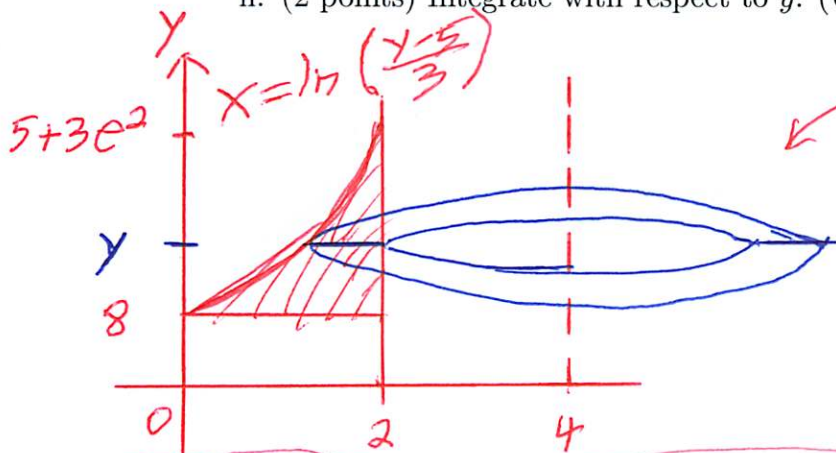
cylinder

$$V = \int_0^2 (\text{surface area}) dx$$

$$= \int_0^2 2\pi r h dx$$

$$V = \int_0^2 2\pi \underbrace{(4-x)}_{\text{radius}} \underbrace{(5+3e^x-8)}_{\text{height}} dx$$

ii. (2 points) Integrate with respect to y . (Use different integrands in parts i and ii.)



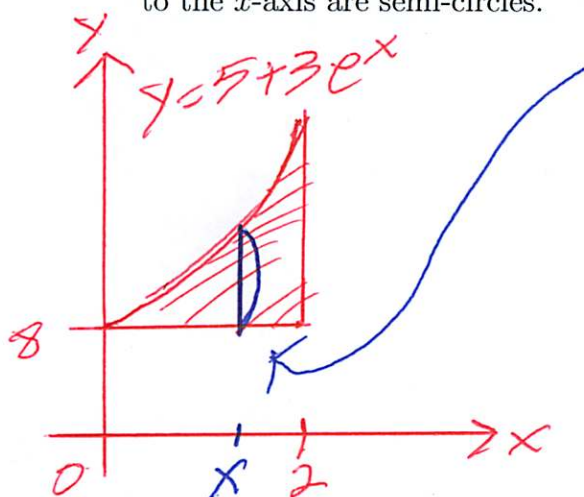
cross-section at y

$$V = \int_8^{5+3e^2} (\text{cross-sectional area}) dy$$

$$= \int_8^{5+3e^2} (\pi r_{\text{out}}^2 - \pi r_{\text{in}}^2) dy$$

$$V = \int_8^{5+3e^2} \left(\pi \underbrace{\left(4 - \ln\left(\frac{y-5}{3}\right)\right)^2}_{r_{\text{out}}} - \pi \underbrace{(4-2)^2}_{r_{\text{in}}} \right) dy$$

- (b) (3 points) The volume of the solid with base \mathbf{R} for which the cross-sections perpendicular to the x -axis are semi-circles.



cross-section at x
is a semi-circle
with area $\frac{1}{2}\pi r^2$
note: $r = \frac{1}{2}$ diameter

$$V = \int_0^2 (\text{cross-section area}) dx$$

$$V = \int_0^2 \frac{1}{2}\pi \left(\frac{\frac{1}{2}(5+3e^x)}{8} \right)^2 dx$$

radius at x

2. (3 points) Find the average value of the function $f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$ on the interval $[25, 81]$.

$$f_{\text{ave}} = \frac{1}{81-25} \int_{25}^{81} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$= \frac{1}{56} \int_{25}^{81} e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}} dx$$

$$= \frac{1}{56} \int_{\sqrt{25}}^{\sqrt{81}} e^u \cdot 2 du$$

$$= \frac{1}{28} \int_5^9 e^u du$$

$$= \frac{1}{28} [e^u]_5^9$$

$$f_{\text{ave}} = \frac{1}{28} (e^9 - e^5)$$

let $u = \sqrt{x}$
 $du = \frac{1}{2} x^{-1/2} dx$
 $2 du = \frac{1}{\sqrt{x}} dx$