Math 220 (section AD?) **Quiz 10** **Fall 2014**

Name: Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

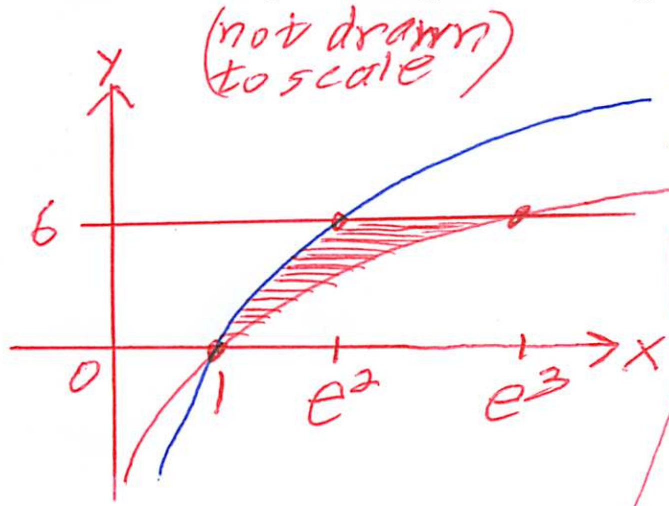
1. (2 points) Precisely state *The Mean Value Theorem.*

Suppose the following conditions hold.

1. f is continuous on $[a,b]$
2. f is differentiable on (a,b)

Then there is a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

2. (2 points) Let **R** be the finite region bounded by $y = 2\ln x$, $y = 3\ln x$ and $y = 6$. Determine the area of **R** by evaluating a definite integral with respect to y.



$y = 3\ln x \Rightarrow x = e^{\frac{1}{3}y}$

$y = 2\ln x \Rightarrow x = e^{\frac{1}{2}y}$

$$A = \int_0^6 \left(e^{\frac{1}{2}y} - e^{\frac{1}{3}y}\right)dy$$

$$= \int_0^6 e^{\frac{1}{2}y}\,dy - \int_0^6 e^{\frac{1}{3}y}\,dy$$

$$= \int_0^3 2e^u\,du - \int_0^2 3e^w\,dw$$

(sub: $u = \frac{1}{2}y$) (sub: $w = \frac{1}{3}y$)

$$= \left[2e^u\right]_0^3 - \left[3e^w\right]_0^2$$

$$= \left[2e^3 - 2\right] - \left[3e^2 - 3\right]$$

$$= 2e^3 - 3e^2 + 1$$

intersection

$3\ln x = 6 \Rightarrow x = e^2$

$2\ln x = 6 \Rightarrow x = e^3$

3. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_{-2}^{0} \frac{42x}{(x^2+3)^2}\,dx = \int_7^3 \frac{21\,du}{u^2} = \int_7^3 21u^{-2}\,du$$

$$= \left[-21u^{-1}\right]_7^3$$

$$= \left(\frac{-21}{3}\right) - \left(\frac{-21}{7}\right)$$

$$= -7-(-3)$$

$$= -4$$

$u = x^2+3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$21\,du = 42x\,dx$

at $x=-2$, $u=(-2)^2+3=7$

at $x=0$, $u=0^2+3=3$

4. (2 points each) Evaluate the indefinite integrals.

(a) $\int 45x^2 \tan^4\left(x^3\right) \sec^2\left(x^3\right)\, dx = \int 15\tan^4(u)\sec^2(u)\,du$

$u=x^3$

$du = 3x^2\,dx$

$15\,du = 45x^2\,dx$

$w = \tan(u)$

$dw = \sec^2(u)\,du$

$15\,dw = 15\sec^2(u)\,du$

$$= \int 15w^4\,dw$$

$$= 3w^5 + C$$

$$= 3\tan^5(u) + C$$

$$= 3\tan^5(x^3) + C$$

(b) $\int \frac{120x^4}{4x^{10}+1}\,dx = \int \frac{120x^4}{(2x^5)^2+1}\,dx = \int \frac{12\,du}{u^2+1}$

$u = 2x^5$

$du = 10x^4\,dx$

$12\,du = 120x^4\,dx$

$$= 12\int \frac{1}{u^2+1}\,du$$

$$= 12\arctan(u)+1$$

$$= 12\arctan(2x^5)+C$$