Math 220 (section AD?) Quiz 1 Fall 2014

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) Determine the domain of the given function.

$$f(x) = \frac{\sqrt{25 - x^2} + \sqrt{9 + \sin(x-1)}}{x^2 - 8x - 20}$$

$$= \frac{\sqrt{25 - x^2} + \sqrt{9 + \sin(x-1)}}{(x+2)(x-10)}$$

To be certain we do not take the square root of a negative number or divide by 0, we must have

$25 - x^2 \geq 0$, $9 + \sin(x-1) \geq 0$, $x \neq -2$, and $x \neq 10$

$x^2 \leq 25$

$\sqrt{x^2} \leq \sqrt{25}$

$|x| \leq 5$

$-5 \leq x \leq 5$

$\sin(x-1) \geq -9$

always true since

$-1 \leq \sin(x-1) \leq 1$

The domain of f is

$[-5, -2) \cup (-2, 5]$

2. (3 points) Given ~~an~~ acute angle θ for which $\sec(\theta) = 8$, evaluate the following quantities.

(a) $\cos(\theta) = \frac{1}{8}$

$$8 = \sec(\theta) = \frac{1}{\cos(\theta)} \Rightarrow \cos(\theta) = \frac{1}{8}$$

(b) $\sin(\theta) = \frac{\sqrt{63}}{8} = \frac{3\sqrt{7}}{8}$

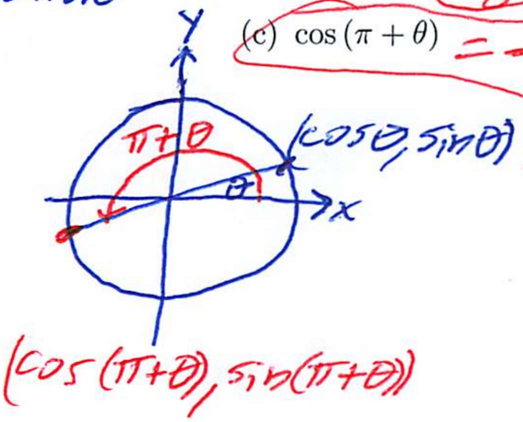
$$\cos^2\theta + \sin^2\theta = 1 \Rightarrow \left(\frac{1}{8}\right)^2 + \sin^2(\theta) = 1$$

$$\Rightarrow \sin^2\theta = 1 - \frac{1}{64} = \frac{63}{64}$$

$$\Rightarrow \sin\theta = \pm\sqrt{\frac{63}{64}}$$

(use $\sqrt{\frac{63}{64}} = \frac{\sqrt{63}}{8}$ since θ acute)

unit circle



(c) $\cos(\pi + \theta) = -\cos(\theta) = -\frac{1}{8}$

or from appendix D,

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(\pi+\theta) = \cos(\pi)\cos(\theta) - \sin(\pi)\sin(\theta)$$

$$= -1 \cdot \cos(\theta) - 0 \cdot \sin\theta$$

$$= -\cos(\theta) = -\frac{1}{8}$$

3. (3 points) Suppose that $f(x)$ is an odd function. If $g(x) = x^4 \sin(f(x))$, then is $g(x)$ an odd function, an even function or neither? Give a very clear justification for your answer.

$$g(x) = x^4 \sin(f(x))$$

$$g(-x) = (-x)^4 \sin(f(-x))$$

$$= x^4 \sin(-f(x)) \quad \text{since } f \text{ is odd}$$

$$= x^4 \cdot -\sin(f(x)) \quad \text{since } \sin \text{ is odd.}$$

$$= -x^4 \sin(f(x))$$

$$= -g(x)$$

Therefore, g is an odd function.