Math 220 (section AD?) **Quiz 9** **Fall 2013**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) A town currently has a population of 8000 people. Suppose the town's population grows by $6t^2$ people per year where t is measured in years from now. What will the town's population be in 10 years?

population in 10 years = current population + change in population

$$= 8000 + \int_0^{10} 6t^2\,dt$$

$$= 8000 + 2t^3\Big|_0^{10}$$

$$= 8000 + \left[2(10)^3 - 2(0)^3\right]$$

$$= 8000 + 2000$$

$$= 10000 \text{ people}$$

another solution: solve differential equation $P'(t) = 6t^2$, $P(0) = 8000$ to get $P(t) = 2t^3 + 8000 \Rightarrow P(10) = 10000$ people

2. (2 points) Rounded off to one place after the decimal, the given definite integral is equal to one of the choices below. Circle the correct choice and show sufficient work to justify your answer. Hint: Obtain an approximation without finding an antiderivative or a limit.

$$\int_1^3 \sqrt{16 + 9\cos^6(e^x + 3)}\,dx$$

(a) 1.4	(b) 3.6	(c) 5.2	(d) 6.1	(e) 8.7
(f) 11.0	(g) 12.5	(h) 15.3	(i) 16.9	(j) 19.8

$$-1 \le \cos(e^x+3) \le 1 \Rightarrow |\cos(e^x+3)| \le 1 \Rightarrow 0 \le \cos^6(e^x+3) \le 1$$

$$\Rightarrow 0 \le 9\cos^6(e^x+3) \le 9 \Rightarrow 16 \le 16 + 9\cos^6(e^x+3) \le 25$$

$$\Rightarrow 4 \le \sqrt{16+9\cos^6(e^x+3)} \le 5$$

$$\Rightarrow \int_1^3 4\,dx \le \int_1^3 \sqrt{16+9\cos^6(e^x+3)}\,dx \le \int_1^3 5\,dx$$

$$\Rightarrow 8 \le \int_1^3 \sqrt{16+9\cos^6(e^x+3)}\,dx \le 10$$

3. (2 points) Evaluate the following indefinite integral.

$$\int \frac{\cos(2\theta)+\sin^2\theta}{\cos\theta}\,d\theta = \int \frac{\cos^2\theta-\sin^2\theta+\sin^2\theta}{\cos\theta}\,d\theta$$

$$= \int \frac{\cos^2\theta}{\cos\theta}\,d\theta$$

$$= \int \cos\theta\,d\theta$$

$$= \sin\theta + C$$

4. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_1^8 \frac{\sqrt[3]{x}-6x}{3x}\,dx = \int_1^8 \left(\frac{x^{1/3}}{3x} - \frac{6x}{3x}\right)dx = \int_1^8 \left(\frac{1}{3}x^{-2/3} - 2\right)dx$$

$$= \left(\frac{1}{3}\cdot\frac{1}{1/3}x^{1/3} - 2x\right)\Big|_1^8$$

$$= \left(x^{1/3} - 2x\right)\Big|_1^8$$

$$= \left((8)^{1/3} - 2(8)\right) - \left((1)^{1/3} - 2(1)\right)$$

$$= (-14) - (-1) = -13$$

5. (2 points) Suppose $w(x) = \int_{\ln x}^{4} \frac{6}{t^3+8}\,dt$. Find $w'(x)$.

$$w(x) = -\int_4^{\ln x} \frac{6}{t^3+8}\,dt$$

$$w'(x) = -\frac{6}{(\ln x)^3+8}\cdot(\ln x)'$$

$$= \frac{-6}{(\ln x)^3+8}\cdot\frac{1}{x}$$