Math 220 **Quiz 8 (take-home)** **Fall 2013**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Sarah Loeb / Hannah Spinoza
- ▷ **AD2**, TR 9:00-10:50, M.Tip Phaovibul
- ▷ **AD3**, TR 1:00-2:50, Cara Monical
- ▷ **ADA**, TR 8:00-8:50, Nima Rasekh
- ▷ **ADB**, TR 9:00-9:50, Hong Liu
- ▷ **ADC**, TR 10:00-10:50, Hong Liu
- ▷ **ADD**, TR 11:00-11:50, Stephen Berning
- ▷ **ADE**, TR 12:00-12:50, Stephen Berning
- ▷ **ADF**, TR 1:00-1:50, Christopher Bailey
- ▷ **ADG**, TR 2:00-2:50, Christopher Bailey
- ▷ **ADH**, TR 3:00-3:50, Neriman Tokcan
- ▷ **ADI**, TR 4:00-4:50, Neriman Tokcan
- ▷ **ADJ**, TR 9:00-9:50, Nima Rasekh
- ▷ **ADK**, TR 10:00-10:50, Michael Obiero Oyengo
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADM**, TR 12:00-12:50, Benjamin Wright
- ▷ **ADN**, TR 1:00-1:50, Benjamin Wright
- ▷ **ADO**, TR 2:00-2:50, Vanessa Rivera-Quiñones
- ▷ **ADP**, TR 3:00-3:50, Vanessa Rivera-Quiñones
- ▷ **ADR**, TR 9:00-9:50, Michael Santana
- ▷ **ADS**, TR 12:00-12:50, Andrew McConvey
- ▷ **ADT**, TR 2:00-2:50, Alessandro Gondolo
- ▷ **ADU**, TR 3:00-3:50, Alessandro Gondolo

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- The quiz should be submitted to your TA at the beginning of your official discussion period on Tuesday, November 5th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Tuesday.**

1. (2 points) The acceleration due to gravity near the surface of some planet is $-10\ m/s^2$. An object is shot upward from the surface of this planet and 8 seconds later it has fallen back to the surface. What is the velocity of this object 2.5 seconds after being shot?

accel $s''(t) = -10$ vel) $s'(t) = -10t + C$ pos.) $s(t) = -5t^2 + Ct + D$

$s(0) = 0 \Rightarrow -5(0)^2 + C(0) + D = 0 \Rightarrow D = 0$

$s(t) = -5t^2 + Ct$

$s(8) = 0 \Rightarrow -5(8)^2 + C(8) = 0 \Rightarrow C = \frac{5(8)^2}{8} = 40$

$s(t) = -5t^2 + 40t$ and $s'(t) = -10t + 40$

$s'(2.5) = -10(2.5) + 40 = 15\ m/s$

2. (2 points) Find a formula for $w(t)$ given that $w''(t) = 5\cos t - 6e^t + 20t - 8$, $w'(0) = 4$ and $w(0) = 2$.

$w''(t) = 5\cos(t) - 6e^t + 20t - 8$

$w'(t) = 5\sin(t) - 6e^t + 10t^2 - 8t + C$

$w'(0) = 4 \Rightarrow 5\sin(0) - 6e^0 + 10(0)^2 - 8(0) + C = 4 \Rightarrow C = 10$

$w'(t) = 5\sin(t) - 6e^t + 10t^2 - 8t + 10$

$w(t) = -5\cos(t) - 6e^t + \frac{10}{3}t^3 - 4t^2 + 10t + D$

$w(0) = 2 \Rightarrow -5\cos(0) - 6e^0 + \frac{10}{3}(0)^3 - 4(0)^2 + 10(0) + D = 2$

$\Rightarrow D = 13$

$w(t) = -5\cos(t) - 6e^t + \frac{10}{3}t^3 - 4t^2 + 10t + 13$

3. (2 points) Suppose that g is continuous at all real numbers, $\int_2^8 g(x)\,dx = 30$ and $\int_2^{10} g(x)\,dx = 42$.

What is the value of $\int_{10}^{8} (4g(x) + 5)\,dx$?

$$\int_{10}^{8}(4g(x)+5)dx = 4\int_{10}^{8} g(x)dx + \int_{10}^{8} 5\,dx$$

$$= 4\left[\int_{10}^{2} g(x)dx + \int_{2}^{8} g(x)dx\right] + \int_{10}^{8} 5\,dx$$

$$= 4\left[-\int_{2}^{10} g(x)dx + \int_{2}^{8} g(x)dx\right] + 5x\Big|_{10}^{8}$$

$$= 4[-42 + 30] + 5\cdot 8 - 5\cdot 10$$

$$= -58$$

(there are other approaches)

4. (2 points) Evaluate the following limit. Use proper notation throughout your evaluation of this limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{14}{n^3} + \frac{12k^2}{n^3} + \frac{4k}{n^2} + \frac{3}{n}\right) = \lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{14}{n^3} + \sum_{k=1}^{n}\frac{12k^2}{n^3} + \sum_{k=1}^{n}\frac{4k}{n^2} + \sum_{k=1}^{n}\frac{3}{n}\right]$$

$$= \lim_{n\to\infty}\left[\frac{14}{n^3}\cdot\sum_{k=1}^{n} 1 + \frac{12}{n^3}\cdot\sum_{k=1}^{n} k^2 + \frac{4}{n^2}\cdot\sum_{k=1}^{n} k + \frac{3}{n}\cdot\sum_{k=1}^{n} 1\right]$$

$$= \lim_{n\to\infty}\left[\frac{14}{n^3}\cdot n + \frac{12}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{4}{n^2}\cdot\frac{n(n+1)}{2} + \frac{3}{n}\cdot n\right]$$

$$= \lim_{n\to\infty}\left[\frac{14}{n^2} + \frac{2(n+1)(2n+1)}{n^2} + \frac{2(n+1)}{n} + 3\right]$$

$$= \lim_{n\to\infty}\left[\frac{14}{n^2} + \frac{4n^2+6n+2}{n^2} + \frac{2n+2}{n} + 3\right]$$

$$= \lim_{n\to\infty}\left[\frac{14}{n^2} + 4 + \frac{6}{n} + \frac{2}{n^2} + 2 + \frac{2}{n} + 3\right]$$

$$= 0 + 4 + 0 + 0 + 2 + 0 + 3$$

$$= 9$$

there are other ways to get this more quickly

5. (2 points) At time t seconds, the velocity of an object is t^3 ft/s. The distance in feet traveled by this object from $t = 12$ to $t = 20$ can be written as a limit of Riemann sums in many different ways. I have shown how to do this for two of the six ways indicated below. Fill in the missing information for the remaining limits so that the only variables appearing are n and k. Do not evaluate these limits.

(a) Using a limit of right Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(12 + k \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$

(b) Using a limit of right Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(12 + (k+1) \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$

(c) Using a limit of left Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(12 + (k-1) \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$

(d) Using a limit of left Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(12 + k \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$

(e) Using a limit of midpoint Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=1}^{n} \left[\left(12 + \left(k - \frac{1}{2}\right) \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$

(f) Using a limit of midpoint Riemann sums,

$$DISTANCE = \lim_{n\to\infty} \sum_{k=0}^{n-1} \left[\left(12 + (k + 0.5) \cdot \frac{8}{n}\right)^3 \cdot \frac{8}{n} \right]$$