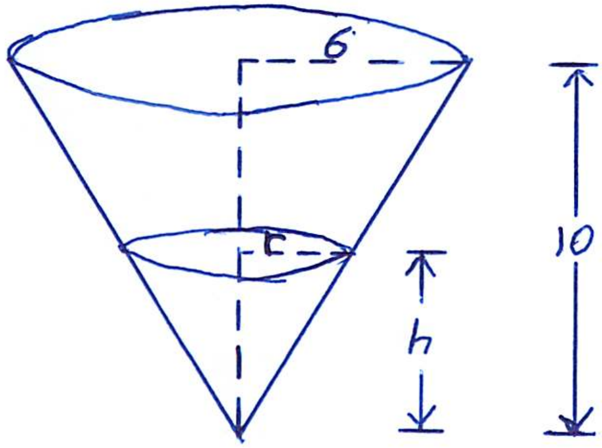
Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A cone-shaped coffee filter of radius 6 *cm* and depth 10 *cm* contains water, which drips out through a hole at the bottom at a constant rate of 1.5 cm^3 per second. How quickly is the water level decreasing when the depth is 8 *cm* ?



given) $\frac{dV}{dt} = -1.5 \ \frac{cm^3}{s}$

want) $\left.\frac{dh}{dt}\right|_{h=8\,cm}$

From similar triangles,
$\frac{r}{h} = \frac{6}{10} \Rightarrow r = \frac{3}{5}h$

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{3}{5}h\right)^2 h$$
$$V = \frac{3\pi}{25}h^3$$
$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{3\pi}{25}h^3\right)$$
$$\frac{dV}{dt} = \frac{9\pi}{25}h^2\frac{dh}{dt}$$
$$-1.5 = \frac{9\pi}{25}(8)^2\left.\frac{dh}{dt}\right|_{h=8\,cm}$$
$$\left.\frac{dh}{dt}\right|_{h=8\,cm} = \frac{-1.5}{\frac{9\pi}{25}(8)^2} = \frac{-25}{384\pi}\ cm/s$$

At a depth of 8 cm, the water level is decreasing by $\frac{25}{384\pi}$ cm/s

2. (4 points) Solve the following differential equations given that the graph of each solution goes through the point $(1, 8)$. You must use the given variables.

(a) $\frac{dw}{dq} = 10q$ $\Rightarrow w = 5q^2 + C$

$8 = 5(1)^2 + C \Rightarrow C = 3$

$w = 5q^2 + 3$

(b) $\frac{dw}{dq} = 6w$ $\Rightarrow w = Ce^{6q}$

$8 = Ce^{6(1)} \Rightarrow C = \frac{8}{e^6}$

$w = \frac{8}{e^6} \cdot e^{6q} = 8e^{6q-6}$

3. (3 points) A bullet is shot upward from the surface of some planet. Between the time that the bullet is shot and the time that the bullet hits the ground, the bullet's height is given by the formula $s(t) = 330t - 5.5t^2$, where t is the number of seconds since the bullet is shot and $s(t)$ is measured in meters above the planet's surface. How many seconds does it take until the bullet reaches its maximum height?

position: $s(t) = 330t - 5.5t^2$

velocity: $s'(t) = 330 - 11t$

At its maximum height, the velocity is 0.

$0 = 330 - 11t$

$11t = 330$

$t = 30$ seconds

Note: The physics of this problem guarantees we have a max. when $s'(t) = 0$. One could also use the first or second derivative tests.