Math 220 (section AD?) **Quiz 5** **Fall 2013**

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Given $h(t) = \arctan\left(t^3\right)$, find its second derivative $h''(t)$.

$$h'(t) = \frac{1}{(t^3)^2+1} \cdot (t^3)'$$

$$= \frac{3t^2}{t^6+1}$$

$$h''(t) = \frac{(3t^2)'(t^6+1) - (3t^2)(t^6+1)'}{(t^6+1)^2}$$

$$= \frac{6t(t^6+1) - 3t^2(6t^5)}{(t^6+1)^2} = \frac{-12t^7+6t}{(t^6+1)^2}$$

2. (2 points each) Compute $\frac{dy}{dx}$ given each of the following equations.

(a) $y = \sqrt{x^6+9}$

$$y = (x^6+9)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(x^6+9)^{-1/2} \cdot \frac{d}{dx}(x^6+9)$$

$$= \frac{1}{2}(x^6+9)^{-1/2} \cdot 6x^5$$

$$= \frac{3x^5}{\sqrt{x^6+9}}$$

(b) $y = \tan^5\left(x^3+2x+8\right)$ $= (\tan(x^3+2x+8))^5$

$$\frac{dy}{dx} = 5(\tan(x^3+2x+8))^4 \cdot \frac{d}{dx}(\tan(x^3+2x+8))$$

$$= 5\tan^4(x^3+2x+8) \cdot \sec^2(x^3+2x+8) \cdot \frac{d}{dx}(x^3+2x+8)$$

$$= 5\tan^4(x^3+2x+8) \cdot \sec^2(x^3+2x+8) \cdot (3x^2+2)$$

(c) $y = \left(x^2+5\right)^{x^3}$

$$\ln(y) = \ln\left(\left(x^2+5\right)^{x^3}\right) = x^3 \cdot \ln(x^2+5)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(x^3 \cdot \ln(x^2+5)\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(x^3)\cdot\ln(x^2+5) + (x^3)\cdot\frac{d}{dx}(\ln(x^2+5))$$

$$= 3x^2 \cdot \ln(x^2+5) + x^3 \cdot \frac{1}{x^2+5}\cdot\frac{d}{dx}(x^2+5)$$

$$= 3x^2 \cdot \ln(x^2+5) + x^3 \cdot \frac{1}{x^2+5}\cdot 2x$$

$$\frac{dy}{dx} = y \cdot \left(3x^2 \ln(x^2+5) + \frac{2x^4}{x^2+5}\right)$$

$$= (x^2+5)^{x^3}\left(3x^2 \ln(x^2+5) + \frac{2x^4}{x^2+5}\right)$$

(d) $\sin\left(x^2y^3\right) = 4x + 2y$

$$\frac{d}{dx}\left(\sin(x^2y^3)\right) = \frac{d}{dx}(4x+2y)$$

$$\cos(x^2y^3)\cdot\frac{d}{dx}(x^2y^3) = 4 + 2\cdot\frac{dy}{dx}$$

$$\cos(x^2y^3)\cdot\left(\frac{d}{dx}(x^2)\cdot(y^3) + (x^2)\cdot\frac{d}{dx}(y^3)\right) = 4 + 2\cdot\frac{dy}{dx}$$

$$\cos(x^2y^3)\cdot\left(2x\cdot y^3 + x^2\cdot 3y^2\cdot\frac{dy}{dx}\right) = 4 + 2\cdot\frac{dy}{dx}$$

$$2xy^3\cos(x^2y^3) + 3x^2y^2\cos(x^2y^3)\cdot\frac{dy}{dx} = 4 + 2\cdot\frac{dy}{dx}$$

$$3x^2y^2\cos(x^2y^3)\cdot\frac{dy}{dx} - 2\cdot\frac{dy}{dx} = 4 - 2xy^3\cos(x^2y^3)$$

$$\frac{dy}{dx}\left(3x^2y^2\cos(x^2y^3) - 2\right) = 4 - 2xy^3\cos(x^2y^3)$$

$$\frac{dy}{dx} = \frac{4 - 2xy^3\cos(x^2y^3)}{3x^2y^2\cos(x^2y^3) - 2}$$