Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate $\sec(\arctan(\pi))$.

① Let $\theta = \arctan(\pi)$
Then $\tan\theta = \pi$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ (actually $0 < \theta < \frac{\pi}{2}$ since $\tan\theta > 0$)

②

$\tan\theta = \pi = \frac{\pi}{1}$ $\left(\frac{\text{opp}}{\text{adj}}\right)$
Pythagorean Theorem $\Rightarrow$

③ $\sec(\arctan(\pi)) = \sec(\theta) = \overset{\text{hyp/adj}}{\frac{\sqrt{\pi^2+1}}{1}} = \sqrt{\pi^2+1}$

2. (2 points) Find all horizontal and vertical asymptotes on the graph of $f(x) = \dfrac{3+2\ln x}{5-6\ln x}$

The denominator equals 0 when $5 - 6\ln x = 0$
$\Rightarrow \ln x = 5/6$
$\Rightarrow x = e^{5/6}$

Also we only plug positive values into $\ln x$.
Thus the domain of $f(x)$ is $(0, e^{5/6}) \cup (e^{5/6}, \infty)$

The only horizontal asymptote occurs at
$$y = \lim_{x\to\infty} \frac{3+2\ln x \nearrow \infty}{5-6\ln x \searrow -\infty} = \lim_{x\to\infty} \frac{3+2\ln x}{5-6\ln x}\cdot\frac{1/\ln x}{1/\ln x} = \lim_{x\to\infty} \frac{3/\ln x + 2}{5/\ln x - 6} = \frac{2}{-6} = -\frac{1}{3}$$

There is a vertical asymptote at $x = e^{5/6}$
since $\lim_{x\to e^{5/6}+} \dfrac{3+2\ln x \to 14/3}{5-6\ln x \searrow 0^-} = -\infty$

$$\lim_{x\to 0+} \frac{3+2\ln x \nearrow -\infty}{5-6\ln x \searrow \infty} = \lim_{x\to 0+} \frac{3+2\ln x}{5-6\ln x}\cdot\frac{1/\ln x}{1/\ln x} = \lim_{x\to 0+} \frac{3/\ln x + 2}{5/\ln x - 6} = \frac{2}{-6} = -\frac{1}{3}$$

Thus there is no vertical asymptote at $x = 0$

3. (2 points each) Evaluate the following limits.

(a) $\lim_{x\to 5} \frac{x^2-3x-10}{x^2-25}$ $\nearrow 0$ $\searrow 0$ (indeterminate form)

$$\lim_{x\to5}\frac{x^2-3x-10}{x^2-25} = \lim_{x\to5}\frac{(x-5)(x+2)}{(x-5)(x+5)}$$
$$= \lim_{x\to5}\frac{x+2}{x+5}$$
$$= \frac{7}{10}$$

(b) $\lim_{x\to 2^+} \frac{e^x}{\ln(5-x^2)}$ $\to e^2$ (pos.) $\to 0^-$ (neg.)

$$\lim_{x\to2^+}\frac{e^x}{\ln(5-x^2)} = -\infty$$

(c) $\lim_{x\to 1} \frac{2-2x}{3-\sqrt{x+8}}$ $\nearrow 0$ $\searrow 0$ (indeterminate form)

$$\lim_{x\to1}\frac{2-2x}{3-\sqrt{x+8}}\cdot\frac{(3+\sqrt{x+8})}{(3+\sqrt{x+8})} = \lim_{x\to1}\frac{2(1-x)(3+\sqrt{x+8})}{(3)^2-(\sqrt{x+8})^2}$$
$$= \lim_{x\to1}\frac{2(1-x)(3+\sqrt{x+8})}{9-(x+8)}$$
$$= \lim_{x\to1}\frac{2(1-x)(3+\sqrt{x+8})}{1-x}$$
$$= \lim_{x\to1} 2(3+\sqrt{x+8})$$
$$= 2(3+\sqrt{1+8}) = 12$$