

Name

Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. An exponential function has a y -intercept of 16 and passes through the point $(-2, 8)$.

(a) (3 points) Determine a formula for this exponential function.

$$y = C \cdot a^x$$

$$(0, 16) \text{ on graph} \Rightarrow 16 = C \cdot a^0 \Rightarrow C = 16$$

$$y = 16 \cdot a^x$$

$$(-2, 8) \text{ on graph} \Rightarrow 8 = 16 \cdot a^{-2} \Rightarrow 8a^2 = 16 \Rightarrow a^2 = 2$$

$$\Rightarrow a = \pm\sqrt{2} \quad (a = \sqrt{2})$$

$$y = 16 \cdot (\sqrt{2})^x$$

or

$$y = 16 \cdot 2^{x/2}$$

(b) (2 points) Find the x -value for the intersection of the graph of this function with the line $y = 160$.

$$160 = 16 \cdot 2^{x/2}$$

$$10 = 2^{x/2}$$

$$\ln(10) = \ln(2^{x/2})$$

$$\ln(10) = \frac{x}{2} \cdot \ln(2)$$

$$x = \frac{2 \ln(10)}{\ln(2)}$$

2. (3 points) Determine all values of x which satisfy the equation below.

$$\ln(\ln(2^x)) = 1$$

$$e^{\ln(\ln(2^x))} = e^1$$

$$\ln(2^x) = e$$

$$x \ln(2) = e$$

$$x = \frac{e}{\ln(2)}$$

3. (2 points) The function $f(x) = 2 + x^3 + 3x^7 + \ln x$ is increasing for $x > 0$. What is the value of $f^{-1}(6)$? Explain your reasoning.

f is increasing $\Rightarrow f$ is one-to-one
 $\Rightarrow f$ has an inverse

$$f(1) = 2 + 1^3 + 3 \cdot 1^7 + \ln(1)$$

$$f(1) = 6$$

Thus $f^{-1}(6) = 1$

since the values of x and y are switched for a function and its inverse