Math 220 **Quiz 12 (take-home)** **Fall 2013**

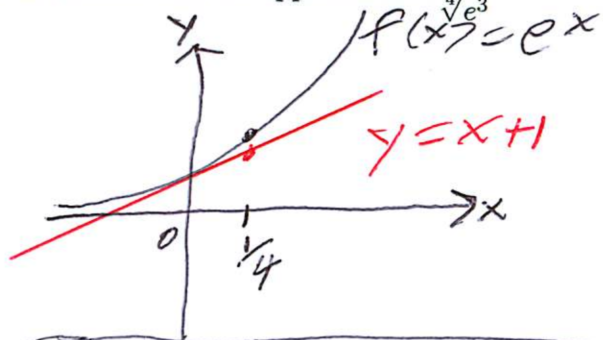
Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 11:00-12:50, Sarah Loeb / Hannah Spinoza
- ▷ **AD2**, TR 9:00-10:50, M.Tip Phaovibul
- ▷ **AD3**, TR 1:00-2:50, Cara Monical
- ▷ **ADA**, TR 8:00-8:50, Nima Rasekh
- ▷ **ADB**, TR 9:00-9:50, Hong Liu
- ▷ **ADC**, TR 10:00-10:50, Hong Liu
- ▷ **ADD**, TR 11:00-11:50, Stephen Berning
- ▷ **ADE**, TR 12:00-12:50, Stephen Berning
- ▷ **ADF**, TR 1:00-1:50, Christopher Bailey
- ▷ **ADG**, TR 2:00-2:50, Christopher Bailey
- ▷ **ADH**, TR 3:00-3:50, Neriman Tokcan
- ▷ **ADI**, TR 4:00-4:50, Neriman Tokcan
- ▷ **ADJ**, TR 9:00-9:50, Nima Rasekh
- ▷ **ADK**, TR 10:00-10:50, Michael Obiero Oyengo
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADM**, TR 12:00-12:50, Benjamin Wright
- ▷ **ADN**, TR 1:00-1:50, Benjamin Wright
- ▷ **ADO**, TR 2:00-2:50, Vanessa Rivera-Quiñones
- ▷ **ADP**, TR 3:00-3:50, Vanessa Rivera-Quiñones
- ▷ **ADR**, TR 9:00-9:50, Michael Santana
- ▷ **ADS**, TR 12:00-12:50, Andrew McConvey
- ▷ **ADT**, TR 2:00-2:50, Alessandro Gondolo
- ▷ **ADU**, TR 3:00-3:50, Alessandro Gondolo

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- **The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, November 22nd.**
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (3 points) Use the techniques of linear approximation found in section 3.10 to approximate $\frac{e}{\sqrt[4]{e^3}}$ without the use of technology.



$$\frac{e}{\sqrt[4]{e^3}} = \frac{e}{e^{3/4}} = e^{1-3/4} = e^{1/4}$$

we find the line tangent to $f(x)=e^x$ at $x=0$.

Point: $(0, f(0)) = (0, e^0) = (0, 1)$

$f'(x) = e^x$

slope: $f'(0) = e^0 = 1$

$y - 1 = 1 \cdot (x - 0)$

$y = x + 1$

$e^x \approx x+1$ for x near 0

$e^{1/4} \approx \frac{1}{4} + 1$

$e^{1/4} \approx 1.25$

Note: From the graphs, we see this is an underestimate

2. (3 points) Let $g(x) = \int_{-125}^{x^3} f(t)\, dt$. Use the techniques of linear approximation found in section 3.10 to approximate $g(5.6)$ given the following information about f.

- f is continuous on the interval $(-\infty, \infty)$
- f is an odd function
- $f(125) = \frac{1}{150}$

$g(5) = \int_{-125}^{5^3} f(t)\,dt = \int_{-125}^{125} f(t)\,dt = 0$ since f is odd and cont.

From F.T.C. (part 1), $g'(x) = f(x^3) \cdot 3x^2$

thus $g'(5) = f(125) \cdot 75$

$= \frac{1}{150} \cdot 75 = \frac{1}{2}$

tangent line at $x=5$

Point: $(5, g(5)) = (5, 0)$

slope: $g'(5) = \frac{1}{2}$

tangent line at $x=5$ is

$y - 0 = \frac{1}{2}(x-5)$

$y = \frac{1}{2}(x-5)$

$g(x) \approx \frac{1}{2}(x-5)$ for x near 5

$g(5.6) \approx \frac{1}{2}(5.6 - 5)$

$g(5.6) \approx 0.3$

3. (4 points) The function $g(x) = x^4 + 3x^2 - 5x$ has precisely one critical number. Determine the value of this critical number using Newton's Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$g'(x) = 4x^3 + 6x - 5$

This derivative exists for all x so the only critical numbers occur when $g'(x) = 0$. We need to solve $4x^3 + 6x - 5 = 0$

Let $f(x) = 4x^3 + 6x - 5$ (thus $f'(x) = 12x^2 + 6$) and apply Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{4x_n^3 + 6x_n - 5}{12x_n^2 + 6}$$

$x_1 = 1$ given as first estimate

$$x_2 = 1 - \frac{4(1)^3 + 6(1) - 5}{12(1)^2 + 6} = \frac{13}{18} \approx 0.7222222222$$

$$x_3 \approx 0.7222222222 - \frac{4(0.7222222222)^3 + 6(0.7222222222) - 5}{12(0.7222222222)^2 + 6}$$

$x_3 \approx 0.653686919 5$

$$x_4 \approx 0.6536869195 - \frac{4(0.6536869195)^3 + 6(0.6536869195) - 5}{12(0.6536869195)^2 + 6}$$

$x_4 \approx 0.6501443634$