1. (6 points) Let $R$ be the finite region bounded by the graphs of $y = \ln x$, $y = 2$, $y = 5$ and $x = 0$. Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

(a) The volume of the solid formed when $R$ is revolved around the horizontal line $y = 6$. Determine this volume in the following two ways.

i. Integrate with respect to $x$.

$$V = \int_{e^2}^{e^5} \left( \pi (6 - 2)^2 - \pi (6 - 5)^2 \right) dx + \int_{e^2}^{e^5} \left( \pi (6 - \ln x)^2 - \pi (6 - 5)^2 \right) dx$$

Note: cross-section is a cylinder, so we used

$$\pi \left( r_\text{out}^2 - r_\text{in}^2 \right) dx$$

ii. Integrate with respect to $y$. (Use different integrands in parts i and ii.)

$$V = \int_{2}^{5} 2\pi r \, dh$$

$$V = \int_{2}^{5} 2\pi (6-y)(e^y) \, dy$$
(b) The volume of the solid with base \( R \) for which the cross-sections perpendicular to the \( x \)-axis are squares.

\[
V = \int_0^e (3)^2 \, dx + \int_1^{e^2} (5 - \ln x)^2 \, dx
\]

2. (4 points) Find the average value of the function \( f(x) = 3x\sqrt{x^2 + 9} \) on the interval \([0,4]\). Simplify your answer.

\[
f_{\text{ave}} = \frac{1}{4-0} \int_0^4 3x\sqrt{x^2 + 9} \, dx
\]

\[
= \frac{3}{4} \int_0^4 \sqrt{u} \cdot \frac{1}{2} \, du
\]

\[
= \frac{3}{8} \int_0^{25} u^{\frac{1}{2}} \, du
\]

\[
= \frac{3}{8} \left[ \frac{1}{3/2} u^{3/2} \right]_9^{25}
\]

\[
= \frac{1}{4} (25)^{3/2} - \frac{1}{4} (9)^{3/2}
\]

\[
= \frac{125}{4} - \frac{27}{4} = \frac{98}{4} = \frac{49}{2} = 24.5
\]