Math 220 (section AD?) **Quiz 11** **Fall 2013**

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (6 points) Let **R** be the finite region bounded by the graphs of $y = \ln x$, $y = 2$, $y = 5$ and $x = 0$. Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

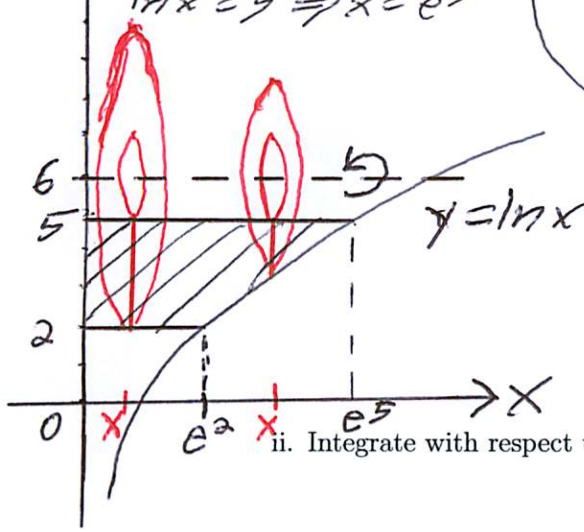
   (a) The volume of the solid formed when **R** is revolved around the horizontal line $y = 6$. Determine this volume in the following two ways.

   i. Integrate with respect to x.

intersection

$\ln x = 2 \Rightarrow x = e^2$

$\ln x = 5 \Rightarrow x = e^5$



$$V = \int_0^{e^2} \left(\pi (6-2)^2 - \pi (6-5)^2\right) dx + \int_{e^2}^{e^5} \left(\pi (6-\ln x)^2 - \pi (6-5)^2\right) dx$$

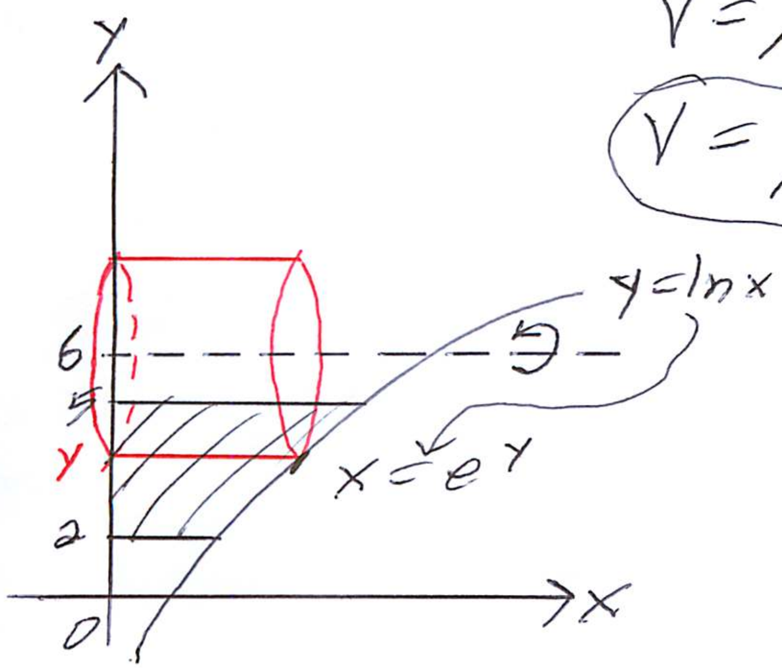
Note: cross-section is

so we used $\int_a^b (\pi r_{out}^2 - \pi r_{in}^2)\, dx$

   ii. Integrate with respect to y. (Use different integrands in parts i and ii.)

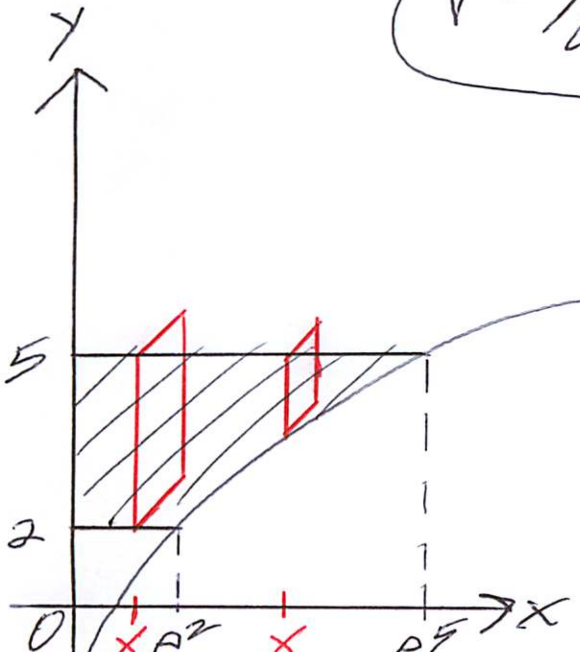
$$V = \int_2^5 2\pi r h \, dy$$

$$V = \int_2^5 2\pi (6-y)(e^y)\, dy$$



(b) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are squares.

$$V = \int_0^{e^2} (3)^2\,dx + \int_{e^2}^{e^5} (5-\ln x)^2\,dx$$



cross-section: □

so we used

$$\int_a^b (\text{side length})^2\,dx$$

2. (4 points) Find the average value of the function $f(x) = 3x\sqrt{x^2+9}$ on the interval $[0,4]$. Simplify your answer.

$$f_{ave} = \frac{1}{4-0}\int_0^4 3x\sqrt{x^2+9}\,dx$$

$$= \frac{3}{4}\int_0^4 x\sqrt{x^2+9}\,dx$$

$u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

x	$u=x^2+9$
0	9
4	25

$$= \frac{3}{4}\int_9^{25} \sqrt{u}\cdot\frac{1}{2}\,du$$

$$= \frac{3}{8}\int_9^{25} u^{1/2}\,du$$

$$= \frac{3}{8}\left[\frac{1}{3/2}u^{3/2}\right]_9^{25}$$

$$= \frac{1}{4}(25)^{3/2} - \frac{1}{4}(9)^{3/2}$$

$$= \frac{125}{4} - \frac{27}{4} = \frac{98}{4} = \frac{49}{2} = 24.5$$